SPIRAL

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United States Patents
1516932-1942026
Other Patents Pending

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Physics 478 - Nuclear Theory

Physics 278 — Nuclear Theory — Prof. Bethe — 2nd Term — 1939-40

Feb. 14

References: Bethe, Rev. Mod. Phys. '36, '37
Stationary States of Nuclei
Theory of Nuclear Reactions
Experimental Results.
Rasetti, Nuclear Physics — '36
Rev. Mod. Phys., '40 Artificial Radioactivity.

Particles occurring in nuclei: Proton, Neutron.
" not " " " : + & − electrons,
mesons of any sort.

For these reasons:

1) If a particle is in a system, collision with another system may eject it; e.g. atomic collisions eject electrons, which are constituents of atoms — Particles ejected from nuclei are always heavy particles (during collisions — electrons only come out afterwards (radioactivity just as light is emitted from atoms (except that when electron is emitted, charge of nucleus changes).

2) Nuclei are very small, electrons very light; By uncertainty principle, electron in space Δx has uncertainty $\geq \frac{\hbar}{\Delta x}$; (relativistically) $E \sim cp \sim \frac{\hbar c}{\Delta x} \sim$ (e.g.) $\frac{10^{-27} \cdot 3 \times 10^{10}}{10^{-12}} \sim 3 \cdot 10^{-5}$ erg

~ 20 m.e.v.

Which is too much; would require a potential barrier of about 10^{9} ev to keep electron in nucleus; such large ones don't exist —

3) Statistical argument —

electrons obey Fermi statistics : $\psi(r_1 r_2 \cdots) = -\psi(r_2 r_1 \cdots)$,

r_1 - spin & space coord of 1st electron.

Pauli principle follows; no 2 electrons with same $n\, l\, m_s\, m_l$ — hence periodic table

Protons, also Fermi stat., spin ½

Deuterons, Bose stat., spin 1

$\psi(r_1 r_2 \cdots) = \psi(r_2 r_1 \cdots)$

If deuteron is composed of p & el., must be: $D = 2P + e$;

But: suppose we have systems A, B, each with n particles, where jA, jB are like particles; if they obey Fermi stat., then:

$$\psi(r_{1A} r_{2A} \cdots r_{nA}; r_{1B}, r_{2B} \cdots r_{nB}) = (-1)^n (r_{1B} r_{2B} \cdots r_{nB}; r_{1A} r_{2A} \cdots r_{nA})$$

But that is equivalent to interchanging the systems;

Hence if n is even, system should obey Bose,
" n " odd, " " " Fermi Stat.

if $D = 2P + e$, n is odd, & D should have Fermi Stat, but it doesn't — in fact it obeys Bose; Hence $D = P + N$

: n even, + all OK.

Feb. 16, 1940

even A, odd Z :

A - mass number.
Z - charge.

Bose Statistics : D, Li^6 B^{10} N^{14}

Proton - electron hypothesis : A protons
A - Z electrons.
total 2A - Z .

Hence nuclei of even Z should obey Bose,
odd Z - - - - Fermi statistics ;

but D, N^{14} , which have odd Z, have Bose stat.;

Proton - Neutron hypothesis :
Z protons
A - Z neutrons
A particles
even A - Bose — observed for D, N^{14} .
Odd A - Fermi - observed for C^{13}

Nuclear charge Z is found from chemical properties ;
or from frequency of X - ray lines
(esp. K_α line : $2p \to 1S$ $\frac{3}{4} R_y Z^2$ + R

Mass Number A : nearest integer to atomic wt. (found from
mass spectrograph with accuracy of 1 in 10^5 for lighter, less for higher[illegible]

Problem of Nuclear Theory:

1) Is description by protons + neutrons sufficient?

e.g. can magnetic moment of nucleus be found by (q-mechanical) vector addition of mm of constituents?

If not, proton + neutrons behave differently from free pr. + neutr.; and we must find theory to account for different properties, bound + free. (In atom, electrons behave the same whether bound or not)

Not much evidence on this. Probably pr-neutr. picture is good within a few percent.

2) What are forces between protons + neutrons? (Most fundamental). Nuclear forces are not electrical, nor gravitational — nothing, in fact, known from macroscopic experiments — In atom, forces were known — but how to describe motion wasn't found till 1926 — Q Mech.
In Nucleus, we know (maybe) the rules of motion — but not forces

3) Is Q. Mech. adequate to describe motion of particles in nucleus?

Bethe thinks likely yes. No evidence to contrary.

Will proceed on assumption ~~that~~ it is.

If forces & mechanisms are known, it is only a mathematical problem to solve problem for eigen values, eigen f'ns etc. — Very complicated, however, ; in atom Hartree approximation can be used; but not in nucleus — a good approx. not found yet.

This not a fundamental question; merely one of calculation

Such Questions:

1) Which approximations can be used in calculation?

No good ones yet — other methods have been used — statistical theory of properties of various ~~levels~~ have been used -

2) Theory of compound Nucleus

Theory of reactions involving complicated nuclei.

Possible to predict accurately prob. of nuclear reaction — in fact, actual cross-sections — with no assumption about forces except ~~that they are~~ large —

Good for predicting exp. results -

Bad for finding real nuclear forces -

Only hope for finding forces, is to investigate simple nuclei. - Simplest nuclei are D (1P, 1N)

Triton T

He^{III}, He^{4} (alpha particle)

But if attempt is made to study $A > 4$, difficulties involved are tremendous, & results don't depend sensitively on forces involved –

Other problems:

Relations between heavy & lighter particles –

1) Mechanism of β-disintegration

To conserve energy, assume neutrino emitted –

Mechanism of theory by Fermi –

Must have a field theory for forces; that's another problem:

2) Field theory of nuclear forces

Properties of proton & neutron — intimately connected with field – (just as properties of electron are closely connected with electromagnetic field).

Field subject to quantization (e.g. photons, for e.m. field)

" of nuclear forces may also give rise to particles –

Greatest progress so far in Theory of comp. nucleus.

Some progress in question of nuclear forces — know approx magnitude & range –

Mechanism of β-disintegration – at least preliminary theory

Pr.-Neut. structure, & applicability of Q.M. – yes, close enough to assume it. Field theory only in beginning stages –

Likewise question of usable approximations — much work done, but not much results.

2/21/40) Nuclear Forces –

Mass no. A
Nuclear charge Z

isotopes : same Z, different A
isobars : same A, different Z

Chemical behavior depends only on Z.
(Very slight dependence on A – only detectable by very refined chemical wa
H, D] separated chemically]
N^{14} N^{15} |

Stable nuclei do not emit $\varepsilon^{\pm}$ – about 300 of them

A : from 1 – 238 (possibly $\frac{239}{93}$).
Z : 1 – 92

Z = 43, 61, 85, 87 have not been observed in stable state
(Masurium) (Illinium) (Florentium) — no serious claims to discovery

Claims to discovery false.

No reason why every element should have a stable representation –

A : 1 – 212 all exist in at least one stable representative
except 5 & 8. $Be^8 \to 2\,He^4$ energy eff. solve
unstable for 5 —
$He^5 \to He^4 + n + 1$ Mev unstable.
Li^5 $He^4 + p + >1$ Mev more "
all others have at least one stable representation.

H	1 2
He	3 4
Li	6 7
Be	(8) 9
B	10 11
C	12 13
N	14 15
O	16 17 18
F	19
P	31
S	32 33 34
Cl	35 37
A	36 38 40
K	39 41 [40]
Ca	40 42 43 44

here even Z has 3 isotopes, odd Z only 1.

A^{40} & Ca^{40} first isobars.

isobars: usually even Z^5, even A.
about 30 (or more) such pairs.

4 years ago:

(stable):

154 isotopes of even A, even Z
52 " " odd A, odd Z
55 " " odd A, even Z
4 " " even A, odd Z.

last 4: $_1H^2$ $_3Li^6$ $_5B^{10}$ $_7N^{14}$
(first four that could occur)

∴ odd A: either even/odd } Z equally likely

even A: always even Z except above 4.

even Z & N — most stable

even Z, odd N } not so stable
odd Z, even N }

odd Z odd N, almost all unstable.

Z protons

$N = A - Z$ neutrons

isotopic number: $A - 2Z = I$

isobars: even A, even Z's differing by 2 units (usually).
about 50 such pairs.

odd A Z's differing by 1 unit very rare.

Radioactive $^{37}Rb^{87}$ $^{38}Sr^{87}$ (spurious ones claimed by Aston
$^{48}Cd^{113}$ $^{49}In^{113}$ disproved by Bainbridge —)
$_{49}In^{115}$ $_{50}Sn^{115}$ (formation of hydrides balls the works)
$_{51}Sb^{123}$ $_{52}Te^{123}$

Canadian mineral containing alkalies, but no alkali earths except Sr — It was suspected that $Rb^{87} \rightarrow Sr^{87} + \bar{e}$ —

Hahn separated Sr from mineral & analyzed ~~it~~ in mass spectrograph — had only Sr^{87} whereas ordinary Sr has 4 isotopes — ∴ Sr from mineral did come from Rb decay)

Rb^{87} has very long lifetime .. must be comparable to age of earth (else there wouldn't be any left). estimated 10^{10} years half-life.

Large difference in nuclear spin between radioactive element & product means long life —

Assume these isobars are radioactive with very long life —

.. No ~~stable~~ isobars with $\Delta Z = 1$ exist;

∴ In general if we have a nucleus A, predict that probably no isobar with ΔZ 1. more likely one of $\Delta Z = 2$

⋮

2/28/40

276 natural nuclei $A \leq 209$

109 odd A		167 even A.	
105 possible values of A		104 possible values of A	
1 not occupied	5	1 not occupied	8
99 singly "		45 singly occupied	
5 doubly "	87 113 115 123 187	52 doubly "	
		6 triply "	
54 even Z		161 even Z	(40, 96, 124, 130, 136, 176)
55 odd Z			

$\underline{40}$: ${}^{18}A$ ${}^{19}K \to {}^{20}Ca$ 96 : ${}^{40}Zr$ ${}^{42}Mo$ ${}^{44}Ru$

$\underline{176}$: ${}^{70}Yb$ ${}^{71}Lu \to {}^{72}Hf$.

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2 elements of odd Z ~~that~~ don't exist ($Z \leq 83$)
43, 61.

2 elements of even Z with no ~~odd~~ isotope of odd A
${}^{18}A$, ${}^{58}Ce$

43: 95 ${}^{42}Mo$
97 ${}^{42}Mo$ hence no ;^{43} would decay
99 ${}^{44}Ru$

similarly : 61: 145 ${}^{60}Nd$
147 ${}^{62}Sm$ ^{61} would decay.

Relative abundance : Sn

112	1.1	119	9.8
114	0.8	120	28.5
115	0.4	122	5.5
116	15.5	124	6.8
117	9.1		
118	22.5		

even more abundant - 118 22.5

Energy. Can determine absolute energy content of nucleus by weighing; which can't be done with atoms -

(Weight of ~~atom~~ nucleus − wt. of constituents) c^2 = binding energy.

Conservation of energy has been so often checked in nuclei that we can use it to predict new facts —

Mass spectrograph used to determine mass; gives e/m of particle; ∴ necessary to know degree of ionization, then mass is known.

Either: compare ~~different~~ ~~same~~ approx same ratio e/m A's ~~with same ionization~~;

~~or: " same A's " different "~~.

e.g.: compare $\frac{1}{2}$ D^+ with H_2^+ $\frac{1}{2}$

or $\frac{1}{4}$ He^+ with D_2^+ $\frac{1}{4}$

$\frac{2}{12}$ C^{++} with D_3^+ $\frac{1}{6}$

$\frac{1}{16}$ $C^{12}H_4^+$ O^+ $\frac{1}{16}$

Bainbridge & Jordan 1936-37
Mattauch
Aston

Bonner P.R. 55 696

atomic masses.

nuclear mass + mass of electrons in neutral atom.

n :	1.008 93
H :	1.008 12
^1D :	2.014 72
$^1H^3$:	3.017 04
$^2He^3$:	3.017 01
$^2He^4$:	4.003 88
$^3Li^6$:	6.016 90
He^6 :	6.020 90
⋮	
C^{12} :	12.003 98
O^{16} :	16.000 00
⋮	

~~Atomic~~ masses because it's only way one can tell whether isobars are ~~stable~~ or ~~unstable~~.

<u>Stability of isobars:</u>

suppose we have: Z^A and $(Z+1)^A$

transform into other by (β-disintegration;

$$Z^A \to (Z+1)^A + \beta^- + \nu \text{ (neutrino)}$$

energetically possible if $Z^A > (Z+1)^A + m$ (+ rest mass of neutrino ~0)

then Z^A is ~~unstable~~, $(Z+1)^A$ stable.

($Z^A + Zm > (Z+1)^A + m + Zm$
$\therefore M_{Z^A} > M_{(Z+1)^A}$ when M's are ~~atomic masses~~)

or: $(Z+1)^A \to Z^A + \beta^+ + \nu$ (perhaps).

more likely: $(Z+1)^A + e^-_K \to Z^A + \nu$ (K-electron capture).

if $(Z+1)^A > Z^A + m$

if $(Z+1)^A - m > Z^A$ $\therefore$ (this more likely)

$(M_{Z+1} > M_Z)$

Of two isobars, one with smaller M is stable.

one " larger M will either emit β^-, or absorb electrons — is ~~unstable~~.

H^3 should be ~~unstable~~ : $H^3 \to He^3 + \beta^- + \nu$

He^6 " " " : $He^6 \to Li^6 + \beta^- + \nu$

Some masses can be determined only by disintegration data — e.g. neutron, H^3; He^3 neutron wouldn't work; H^3 unstable, He^3 too rare —

3/1/40 !!

To use disint. data:

1) must know kin. en. of incident particles
2) Measure accurately range of product of reaction
3) Range-energy relation — (from collision theory) from measurements on α-particles; for $\alpha > 12$ Mev, $p > 3$ ... must derive ~~theoretically~~

Liv. + Bethe Rev. Mod. Phys. 9

4) Geometry

must know θ.

$$\underline{p}_{recoil} = \underline{p}_{inc.} - \underline{p}_{product}$$

and

$$p_{rec}^2 = p_{in}^2 + p_{prod}^2 - 2\, p_{in} p_{prod} \cos\theta$$

$$E_{rec} = \frac{p_{rec.}^2}{2M_{rec}} = \frac{M_{in} E_{in} + M_{pro} E_{prod} - 2\sqrt{M_i M_p E_i E_p}\cos\theta}{M_{rec.}}$$

$$Q = E_{pr} + E_{rec} - E_{inc.}$$

Thin targets: ones in which energy loss is within desired accuracy of energy measurement — almost impossible with solid targets; possible with gaseous targets —

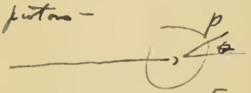
Solid targets must be considered thick; then a very thick one is desirable (correction difficult for medium thick target), so that product particle is completely absorbed; from knowledge of cross-section – energy relation & stuff, can finally calculate Q –

Oliphant & Rutherford '33 P.R.S.

$$ {}^1H^2 + {}^1H^2 = \begin{cases} {}^1H^3 + {}^1H^1 + 3.98 \text{ Me.v.} \pm .01_5 \\ {}^2He^3 + {}^0n^1 + 3.28 \text{ ''} \pm .07 \end{cases} $$

10% – 1.3 Mev – Bonner – but Bethe doesn't believe: because no γ-rays emitted

To measure ↗, must use recoil protons –



$$E_n = \frac{E_p}{\cos^2\theta}$$

Allison – electric deflection

adjust potential so that products of reaction can go through –

good only up to 2 Mev. accurate to a few kilovolts

$$Be^9 + H^1 \rightarrow Be^8 + H^2$$

Binding energy of H^2, from:

$$H^2 + \gamma \rightarrow H^1 + n^1$$

ThC'' 2.62 Mev. | $.22_5$ Mev ± .03 ← same, since γ-ray momentum is negligible – ∴ total: .45 Mev ± .06 –

P.R.S. Chadwick & Goldhaber '3[illegible]
", Feather & Bretscher '3[illegible]

cross section:
Von Halban & Kowarski Nature '38

∴ Binding energy of H^2 : $2.62 - 0.45 = 2.17 \pm .06$ Mev.

From this, find mass of n

From mass spectr. : $2H - H^2 = 1.53$ m.m.u.

$= 1.42$ M.e.v.

$H + n - H^2 = 2.17$

∴ $H - n = -.75$

$n - H = .75$ M.e.v.

Calculate difference in atomic weight of He^3 + H^3.

3/15/40 Wave fn of deuteron over most of space: $e^{-(1-a)\alpha}$

Scattering of neutrons by protons.

Deuteron eq'n: $\frac{d^2u_0}{dr^2} + \left(\frac{mE}{\hbar^2} - \frac{mV}{\hbar^2}\right)u_0 = 0$ $l = 0$

Scattering (Differential) Cross Section:

$$d\sigma = \left|\frac{1}{2ik}\sum_l (e^{2i\delta_l}-1)(2l+1)P_l\cos\theta\right|^2 2\pi\sin\theta$$

$k = \frac{\sqrt{mE}}{\hbar}$ δ_l very small for ordinary neutron energies, $l \neq 0$

distance at which particles pass each other:

$\sim \lambda l$ $\lambda = 1/k$

For $l = 1$, δ_1 small, if $\lambda > a$ a - range of for[ces]
$a \sim 2\times10^{-13}$

$$\lambda = \frac{\hbar}{\sqrt{mE}} = \sqrt{\frac{41.5}{E}\text{ Mev.}}\ .10^{-13}\text{ cm.}$$

$\therefore \lambda > a$ if $E < 10$ Mev

For most scattering expts, $E \sim 2, 4$ Mev;

$\therefore$ only ~~δo~~ δ_0 appreciable

Calculate δ_1 for rectangular potential with range $a = 2\times10^{-13}$ cm

get numerical values for $E_0 = 2$ + for 15 Mev.

E is energy in c. of m. coordinates; $= \frac{1}{2}E_0$ (en. in lab. syst[em])

${}_3Li^7 + {}_1H^2 \to {}_4Be^8 + {}_0n$ 15 Mev

$H^2 + H^2 \to He^3 + n$ 2 Mev

if: only $l = 0$, Spherical symmetry (in c. of m. system)

in lab. system: scattered neutrons as $\sin\theta \cos\theta \, d\theta$.

Experimental difficulties: a) scattered neutrons

b) discrimination against short range protons.

$E_{prot} = E_{neutr} \cos^2\theta$. ← (at large angles)

Cambridge Dee + Gilbert '38 P.R.S.

Illinois Kruger, Green, Shoupp '38 P.R.

Established that spherically sym. scatt. for 2.5 Mev neutrons

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for only δ_0;
$$d\sigma = \left|\frac{1}{2ik}\left(e^{2i\delta_0} - 1\right)\right|^2 2\pi \sin\theta \, d\theta$$

$$= \left|\frac{1}{2ik}\left[1 + 2i\delta_0 - 1\right]\right|^2 2\pi \sin\theta \, d\theta$$

$$= 2\pi \left(\frac{\delta_0}{k}\right)^2 \sin\theta \, d\theta$$

$$\sigma = 4\pi \left(\frac{\delta_0}{k}\right)^2$$

for larger δ_0: $\left|e^{2i\delta_0} - 1\right|^2 = 2(1 - \cos 2\delta_0) = 4 \sin^2 \delta_0$

and $\sigma = 4\pi \left(\frac{\sin\delta_0}{k}\right)^2$.

Rectangular ~~potential~~: $\gamma^2 = \frac{m V_0}{\hbar^2}$

$r < a$: $\frac{d^2 u}{dr^2} + (k^2 + \gamma^2) u = 0$

$u = c_1 \sin\left(\sqrt{k^2 + \gamma^2}\right) r$

$r > a$: $u'' + k^2 u = 0$

$$u = c_2 \sin kr + c_3 \cos kr$$

$$= c_4 \sin(kr + \delta_0)$$

match at $r = a$:
(to find δ_0).

$$\frac{1}{u}\frac{du}{dr} = \sqrt{k^2+\gamma^2} \cot (k^2+\gamma^2)^{1/2} a = k \cot(ka + \delta_0)$$

if $k << \gamma$: $\gamma \cot \gamma a = k \cot(ka + \delta_0)$
(which means $E << V_0$)

Ground state : $\sqrt{\gamma^2 - \alpha^2} \cot \sqrt{\gamma^2-\alpha^2}\, a = -\alpha$

(since $\alpha^2 << \gamma^2$) or $\gamma \cot \gamma a = -\alpha$

$$\therefore \ -\alpha = k \cot(ka + \delta_0)$$

also ka is small ; $ka << \gamma a \sim \frac{\pi}{2}$.

$\therefore$ Neglect it.

$$\therefore \ -\alpha = k \cot \delta_0$$

$$\cot \delta_0 = -\frac{\alpha}{k}.$$

$$\therefore \ \sigma = 4\pi \frac{\sin^2 \delta_0}{k^2}$$

$$= 4\pi \frac{1}{k^2(1+\cot^2\delta_0)}$$

$$= \frac{4\pi}{k^2(1+\frac{\alpha^2}{k^2})} = \frac{4\pi}{k^2+\alpha^2}$$

$$= 4\pi \frac{\hbar^2}{m}\frac{1}{\epsilon + E} = \frac{4\pi\hbar^2}{m(\epsilon + \frac{E_0}{2})}.$$

$$\epsilon = \text{B.E. of deuteron} = \frac{\hbar^2}{m}\alpha^2$$

$$\sigma = \frac{4\pi \cdot 41.5 \text{ Mev}}{\epsilon + \frac{1}{2}E_b} \cdot 10^{-26} = \frac{5 \times 10^{-24}}{2.17} = 2.3 \cdot 10^{-24} \text{ cm}^2$$

if $E_0 << \epsilon$: observed: $15\text{-}18 \cdot 10^{-24} \text{ cm}^2$

∴ Theory doesn't give correct results for slow neutrons, tho it does for fast.

3/20/40 | Cross section for neutrons scattered by protons:

$$\sigma = \frac{4\pi}{\alpha^2 + k^2} = 4\pi . \frac{\hbar^2}{m} \cdot \frac{1}{\varepsilon + \frac{1}{2}E_0}$$

$$E_0 = 0, \quad \sigma = 2.3 \times 10^{-24} \text{ cm}^2.$$

Measurements for small E_0 give $\sigma = 15\text{-}18 \times 10^{-24}$ cm^2

Discrepancy can't be removed by finite range of nuclear forces;

Wigner 1935 gave solution;

p, n each have spin ½;

P + N ∴ has spin 0 or 1.

↓ singlet state ↓ triplet state. (because of splitting in magnetic field)

Ground state: spin = 1, triplet.

∥ spin

orbital momentum = 0, ∴ S state

∴ total ang mom = 1 ∴ Ground state: $^3S_1$

possible α has different value for singlet & triplet states;

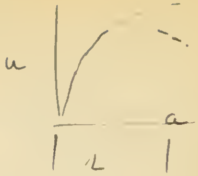
$-\alpha = \frac{1}{u}\frac{du}{dr}$ at $r = a$, for triplet state.

Then $\left(\frac{1}{u}\frac{du}{dr}\right)_{r=a} = \beta$, singlet state;

if $\beta > 0$, u increases at a;

$\beta < a$, u decreases at a;

$\beta > 0$, no stable singlet state; ("virtual" state)



But if $\beta < 0$, outside a, u: decreasing exponential

∴ stable singlet state, $B.E. = \frac{\hbar^2}{m}\beta^2$;
$= \epsilon'$

Then $\sigma_{singlet} = \frac{4\pi}{\beta^2 + k^2}$

$$= 4\pi \frac{\hbar^2}{m} \frac{1}{\epsilon' + \frac{E_0}{2}}$$

If many neutrons are falling on a proton, some will have ∥ spin, some anti-∥; in fact, 3/4 parallel,

& 1/4 anti-p., since weight of ^3S state is 3.

$$\therefore \sigma_{obs} = \frac{\sigma_{singl}}{4} + \frac{3\sigma_{trip}}{4}$$

$$= \pi \frac{\hbar^2}{m}\left(\frac{1}{\epsilon' + \frac{E_0}{2}} + \frac{3}{\epsilon + \frac{E_0}{2}}\right)$$

for $\sigma_{obs} = 16.8 \times 10^{-24}$, $\epsilon' = \frac{1}{25}\epsilon$,

and $\beta' = \frac{1}{5}$ ~~α~~.

(still don't know whether β is + or −)

Interference of neutrons scattered from two protons

two proton, as in H_2^{m} have spins either ∥ or anti-∥.

incoming neutron, also either ‖, or anti-‖;

if ‖, triplet scattering from each; } for ortho H_2,
if anti-‖ singlet " " "; } $(\uparrow\uparrow)$

∴ constructive interference. — large cross-section.

for para H_2 $(\uparrow\downarrow)$, triplet scattering from one & singlet from other. call amplitudes a + b.

interference; scattered amplitude, a + b; if they have same sign, large intensity, different sign, small intensity.

∴ constructive interference if $\beta < 0$ (real singlet state)
destructive " if $\beta > 0$ (virtual state)

More exact treatment.

spin functions $\sigma_{m_s=+\frac{1}{2}}$ $\tau_{m_s=-\frac{1}{2}}$

$$\text{Triplet}: \begin{cases} \sigma(n)\,\sigma(p) = \chi_1 \\ \frac{1}{\sqrt{2}}\left[\sigma(n)\,\tau(p) + \tau(n)\,\sigma(p)\right] = \chi_0 \quad 2 \\ \tau(n)\,\tau(p) = \chi_{-1} \quad 3 \end{cases}$$

$$\text{Singlet}: \frac{1}{\sqrt{2}}\left[\sigma(n)\,\tau(p) - \tau(n)\,\sigma(p)\right] = \chi_s \quad 4$$

Define: scattering operator S

S applied to spin function of original system gives spin function of scattered beam.

$$S \sum_1^4 c_n \chi_n = a \sum_1^4 c_n \chi_n + b c_5 \chi_5 .$$

$$\cot \delta_0 = \frac{\beta}{k} . \qquad \left(\text{scattering} \propto e^{2i\delta_0} - 1\right) .$$

if $\beta > 0$, $\delta_0 \to 0$ as $k \to 0$ and $e^{2i\delta_0} - 1 \to 2i\delta_0 > 0$

if $\beta < 0$, $\delta_0 \to \pi$ as $k \to 0$ and $e^{2i\delta_0} - 1 \to 2i(\delta_0 - \pi) < 0$

$\therefore$ $b > 0$ for $\beta > 0$ (virtual state)

$b < 0$ for $\beta < 0$ (real state)

$a < 0$.

Para H_2 : $\frac{1}{\sqrt{2}}\left(\sigma(1)\tau(2) - \sigma(2)\tau(1)\right)$

$$\chi = \frac{1}{\sqrt{2}}\left[\sigma 1 \tau 2 - \sigma 2 \tau 1\right] \sigma(n) .$$

$$S\chi = \frac{1}{\sqrt{2}}\left[\tau_2 S_1 \sigma(1)\sigma(n) - \sigma_2 S_1 \tau_1 \sigma_n\right]$$

triplet + do the same S_2

$$= \frac{1}{\sqrt{2}}\left[\tau_2 \, a \, \sigma(1)\sigma(n) - \sigma(2)\left\{\tfrac{1}{2}(a+b)\sigma(n)\tau(1) + \tfrac{1}{2}(a-b)\tau(n)\sigma(1)\right.\right.$$

$$+ \sigma(1)\left[\tfrac{1}{2}(a+b)\tau(2)\sigma(n) + \tfrac{1}{2}(a-b)\sigma(2)\tau(n)\right.$$

$$- \tau(1)\sigma(2)\sigma(n)$$

$S \, \tau(1)\sigma(n)$

$$= \frac{1}{\sqrt{2}}\left(a\chi_0 + b\chi_3\right)$$

$$= \tfrac{1}{2}(a+b)\sigma(n)\tau(1) + \tfrac{1}{2}(a-b)\tau(n)\sigma(1)$$

$$= (3a+b)\chi + (a-b)\chi_{\text{triplet}}(1,2) \, {}^{\sigma}_{\tau}(n) .$$

↑ elastic scattering

↑ transition from ortho → para H_2

If energy of neutron is too small to make ortho → para H_2, only elastic scattering will take place;

for a virtual state $b > 0$, $|b| = 5 \times 10^{-13}$,

and $S\chi = -2a\chi$

for real state, $S\chi = 8a\chi$.

Scattering of neutron by para H_2 will be 4 times as large (& same order as ortho H_2 scattering) ~~of~~ for ~~virtual~~ real state as for virtual;

Experiment: small scattering for para H_2;

∴ singlet state is virtual ($b < 0$, $\beta < 0$)

3/22/40 | Scattering depends on state of nucleus.

Scattering by a number of particles bound by forces;

Classically, scattering independent of forces; if time of collision is small compared with periods of system (as it usually is) Then energy is distributed among particles of system.

Q. M'ally ; time is longer - so more complicated -

Particular form of nuclear potential doesn't affect results, since range is so much smaller than wave length of molecular wave fn's ∴ replace it by longer + lower potential (still small relative to molecular wave length) + apply Born approximation.

$$\sigma = \frac{\mu^2}{\pi \hbar^4} \left| \int V \, d\tau \right|^2$$

1) particle ~~tightly~~ bound : $\mu = m \cdot 1$

2) particle free : $\mu = \frac{mM}{m+M} = 1/2$.

∴ cross section of bound particle 4 times as much.

Differential cross section for ~~transition~~ from state m to n ;

$$\sigma_{mn} = \left| \frac{m}{2\pi \hbar^2} \int \psi_m^*(\underline{r}_p) \, e^{-ik'\underline{r}_n} \, V(\underline{r}_p \underline{r}_n) \, \psi_n(\underline{r}_p) \, e^{ik\underline{r}_n} \, d\tau_p \, d\tau_n \right|$$

(labels: ψ_m^* ↓ proton final; $e^{-ik'r_n}$ ↓ neutron; V ← potential; ψ_n ↑ p; e^{ikr_n} ↑ neutr initial state)

cross section for free proton :

Let $V = c\,\delta(\underline{r}_p - \underline{r}_n)$

$$Q = \frac{m^2}{4\pi \hbar^4} c^2$$

$$\therefore V = \frac{\hbar^2}{m} \sqrt{4\pi Q} \; \delta(\underline{r}_p - \underline{r}_n) .$$

$$\therefore c = \frac{\hbar^2}{m} \sqrt{4\pi Q}$$

$$\therefore \sigma_{mn} = \left| \sqrt{\frac{Q}{\pi}} \int \psi_m^*(\underline{r}_p) \, e^{ik''\underline{r}_p} \, \psi_n(\underline{r}_p) \, d\tau_p \right|$$

$k - k' = k''$

$$= \frac{Q}{\pi} \left| \left(e^{ik\underline{r}_p}\right)_{mn} \right|^2$$

(q = cross section / unit solid angle) $= 4q \left| \left(e^{ik\underline{r}_p}\right)_{mn} \right|^2$

If several protons, sum over all of them in molecule -

suppose proton in isotropic oscillator;

$$\psi = \psi_x \psi_y \psi_z$$

$$\psi_0 = e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2} a^{-1/2} \pi^{-1/4}$$

$$\therefore \quad \sigma_{00} = 4q \left| \left(e^{i h \cdot r} \right)_{00} \right|^2$$

$$= 4q \left| \frac{a^{-1}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{i h'' x - \left(\frac{x}{a}\right)^2} dx \right|^2$$

$$= \frac{4qa^2}{\pi} \left| a \int_{-\infty}^{\infty} e^{i b y - y^2} dy \right|^2$$

$h'' = 2h \sin \theta/2$

$b = ah''$

$= 2ah \sin \theta/2$

$$= \frac{4q}{\pi} \cdot \left| \sqrt{\pi} \cdot e^{-\frac{b^2}{4}} \right|^2$$

$$= 4q\, e^{-\frac{b^2}{2}}$$

$$= 4q\, e^{-2(ah)^2 \sin^2 \theta/2}$$

$$\sigma_{obs} = \sigma \cdot \frac{v_{rel} \text{ to part.}}{v_{\text{rel. to observer}}}$$

3/27/40 ← annoying hard!

Photo~~electric~~ effect of D.

cross section for any photo-el. effect:

$$\sigma = \frac{4\pi^2 w}{c} \left| M_{0E}^{el} \right|^2.$$

(imagine light wave disturbing system; calculate ~~transition~~ probability.) Wave fn of excited state normalized per unit energy.

For deuteron: $M_{0E}^{el} = e \int \psi_0 \psi_E \, d\tau \cdot \frac{1}{2} z$

↑ coordinate of proton w.respect to neutron ($\frac{1}{2} z$ – w. resp. to c.g.)

$r > a$: $\psi_0 = \frac{1}{\sqrt{\pi \alpha}} \frac{e^{-\alpha r}}{r}$

$$\psi_E = e^{i \underline{k} \cdot \underline{r}} \cdot \frac{1}{\sqrt{\Omega}} \sqrt{\frac{\Omega \cdot 4\pi p^2 dp}{h^3 dE}}$$

($l = 1$ since only ~~transition~~ to P state will be allowed, on acct of factor z)

might as well write a plane wave

Ω: volume of box;

$\sqrt{\ }$: no of qu. states per unit energy in that box.

but $\frac{dE}{dp} = v$ $\quad p = \frac{1}{2} M v$ ($\frac{1}{2}$ because rel. coords)

$$\therefore \psi_E = e^{i \underline{k} \cdot \underline{r}} \sqrt{\frac{4\pi \cdot p^2 M}{h^3 \cdot 2p}} = e^{i \underline{k} \cdot \underline{r}} \sqrt{\frac{M p}{4\pi^2 \hbar^3}}$$

$$\therefore M_{0E}^{el} = \frac{e \sqrt{M p}}{4\pi \ \hbar^{3/2} \sqrt{\pi \alpha}} \int d\tau \ \frac{e^{-\alpha r}}{r} z \, e^{i \underline{k} \cdot \underline{r}}$$

$$= \qquad \frac{1}{i} \frac{\partial}{\partial k_z} \int d\tau \ \frac{e^{-\alpha r}}{r} e^{i \underline{k} \cdot \underline{r}}$$

$$= 2\pi \int r^2 \, dr \; \frac{e^{-\alpha r}}{r} \left(\frac{e^{ikr}}{ikr} + \text{conj} \right)$$

$$= \frac{2\pi}{ik} \frac{1}{\alpha - ik} + \text{conj}.$$

$$= \frac{2\pi}{ik} \left[\frac{1}{\alpha - ik} - \frac{1}{\alpha + ik} \right]$$

$$= \frac{4\pi i k}{ik(\alpha^2 + k^2)} = \frac{4\pi}{\alpha^2 + k^2}.$$

$$\frac{1}{i} \frac{\partial}{\partial k_3} (\;\;) = -\frac{4\pi}{i} \frac{2k_3}{(\alpha^2 + k^2)^2}$$

$$\therefore M^{el}_{0E} = -\frac{e\sqrt{Mp}}{i\hbar^{3/2}\sqrt{\pi\alpha}} \frac{2k_3}{(\alpha^2 + k^2)^2}$$

$$| \; |^2 = \frac{4e^2 Mp}{\pi\alpha \hbar^3} \frac{1}{3} \frac{k^2}{(\alpha^2 + k^2)^4}$$

average over all directions.

$$\therefore \quad \sigma = \frac{16 e^2 \pi \omega M p}{3c\alpha \hbar^3} \frac{k^2}{(\alpha^2 + k^2)^4}.$$

$$= \frac{2\pi}{3} \cdot \frac{e^2}{\hbar c} \cdot \frac{\hbar^2}{M} \cdot \frac{\sqrt{\epsilon} \, E^{3/2}}{(E + \epsilon)^3}$$

ϵ : B.E. of deuteron

E . en. of

$$= 1.16 \cdot 10^{-26} \cdot \frac{(\gamma - 1)^{3/2}}{\gamma^3}$$

$$\gamma = \frac{\hbar\omega}{\epsilon} = \frac{E + \epsilon}{\epsilon}.$$

Scattering cross-section was:

$$\sigma = \frac{4\pi\hbar^2}{M(\epsilon + E)}$$

(triplet scat.)

γ-rays less than ϵ energy, no effect.

If greater, excess energy goes to kin. en. of p. & neutr.

cross-section has a max when ?

at very high freq., σ decreases again;

$\gamma = \frac{\hbar\omega}{\epsilon}$

$$\sigma_{max} = 1.4_5 \cdot 10^{-27}\ cm^2 .$$

radiation effects are ~~small~~ compared to (only) particle effects;

$\frac{e^2}{\hbar c}$ always comes in; even less than ~~that factor~~, though;

Measurements made at $\hbar\omega = 2.62$ Mev (Th C'' γ's)

$\epsilon = 2.17$,

$\therefore \gamma = 1.2$,

To compare with exp., must consider wave functions in $r < a$ as well (nuclear force range). Multiplies σ by factor of about 2. (Breit + Condon, P.R. 1936).

They get $\sigma = 9 \cdot 10^{-28}$ for Th. C'' γ's,

observed same, by Halban + Kowarski, Nature?

Prob. of ~~emission~~ in ~~a given direction~~ ~~direction~~ of incidence ~~is proportional to~~ a given direction is prop to $\cos^2\theta$,

when Θ is angle betw. polarization of incident radiation, & emitted proton;

For unpolarized γ-rays;

Proton scattering $\sim l_y^2 + l_z^2 \sim \sin^2 \Theta$.

(incident radiation in x-direction)

(Θ = angle betw. x & proton)

(verified - more or less.

Photo-magnetic effect.

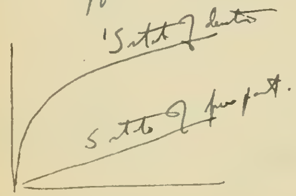
$$\sigma = \frac{4\pi^2 \omega}{c} \left| M_{0E}^{Magn} \right|^2$$

$$M = \sum_{spins} \int \psi_E^* \mu_z \psi_0 \, d\tau$$

μ - magn. moment operator

$\mu = e \frac{\hbar}{Mc}$. $M \sim e a$ $a \gg \frac{\hbar}{Mc}$; $\therefore$ magnetic effects smaller than elec.

photo-mag effect can give a transition from ground S-state to excited S-state, which elec. effect can't.



3/29/40 | $\mu_z = \mu_p \sigma_{zp} + \mu_n \sigma_{zn}$ σ - Pauli operator

$$\mu_p = 2.718$$
$$\mu_n = -1.953$$
nuclear magneton

one " " $= \frac{e\hbar}{2Mc}$.

$$\psi_0 = \frac{e^{-\alpha r}}{\sqrt{\pi\alpha}\, r} \cdot \frac{1}{\sqrt{2}}\left[\sigma(1)\tau(2) + \tau(1)\sigma(2)\right]$$ ($\sigma_1 \sigma_2$, $\tau_1 \tau_2$) — triplet

$$\psi_E = \psi_E(1) \frac{1}{\sqrt{2}}\left[\sigma(1)\tau(2) - \tau(1)\sigma(2)\right]$$ — singlet.

1 - p
2 - n

$$M = \int d\tau\, \psi_0(1)\psi_E(1) \cdot \sum_\sigma \frac{1}{2}\left[\sigma(1)\tau(2) - \tau(1)\sigma(2)\right]\left\{\left[\mu_p - \mu_n\right]\sigma(1)\tau(2) + (\mu_n - \ldots)\,\tau(1)\sigma\ldots\right.$$

$$= \sum_\sigma \frac{1}{2}\left[\chi_{sing}\left[\mu_p - \mu_n\right]\chi_{sing}\right.$$

$=$

$$= (\mu_p - \mu_n)$$

$(\mu_z \chi_{trip} = (\mu_p - \mu_n)\chi\ldots$

Normally, $\int \psi_0 \psi_E = 0$;

But here one is singlet & other triplet; (Both s-states)

Proof of orthogonality rests on equality of pot. en. in the two cases; here forces are entirely different; $\therefore$ ψ_0 & ψ_E are not orthogonal.

$$\psi_{E\,l=0} = \frac{\sin(kr+\delta)}{kr}\sqrt{\frac{4\pi p^2 dp}{h^3 dE}} \quad r > a$$

$$\tan\delta = \frac{k}{\beta}$$

$$\int d\tau = \sqrt{\frac{Mp}{2\pi^2\hbar^3}} \cdot \frac{4\pi}{\sqrt{\pi\alpha}} \frac{1}{k}\int \frac{1}{r^2} e^{-\alpha r}\sin(kr+\delta)\, r^2 dr$$

$$\zeta = \mathrm{Im}\int e^{-\alpha r + ikr + i\delta}\, dr$$

$$= \mathrm{Im}\frac{e^{i\delta}}{\ldots} = \frac{k\cos\delta + \alpha\sin\delta}{\alpha^2 + k^2}$$

$\cos\delta = \frac{\beta}{\sqrt{h^2+\beta^2}}$ $\sin\delta = \frac{h}{\sqrt{h^2+\beta^2}}$

$\therefore \int = \frac{h(\beta+\alpha)}{(\alpha^2+h^2)\sqrt{\beta^2+h^2}}$. $\begin{cases} \frac{6}{5}\alpha & \alpha+|\beta| \text{ if singlet state virtual} \\ \frac{4}{5}\alpha & \alpha-|\beta| \text{ if real.} \end{cases}$

from ~~exp~~, $|\beta| \sim \frac{1}{5}\alpha$; $\therefore$ ↑

$\therefore$ 3/2 as much for virtual state;

and cross-section, 9/4 or over twice as great.

$$\sigma_{virtual} \sim \frac{9}{4}\sigma_{real}$$

Experiment again indicates virtual state.

if trip + singl state have same B.E., $\alpha = \beta$, and $\alpha - |\beta| = 0$;

(for then the wave f'ns are orthogonal).

$g = \mu$

$$\therefore \quad \sigma = \frac{4\pi^2\omega}{c} \cdot \frac{8}{\pi} \cdot \frac{Mp}{\alpha h^3} \frac{(\beta+\alpha)^2}{(\alpha^2+h^2)^2(\beta^2+h^2)} e^2\left(\frac{h}{2Mc}\right)^2(g_p - g_n)$$

4.7

$$\alpha^2 + h^2 \sim \omega$$

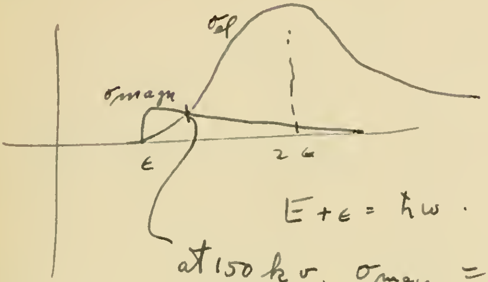
$$\sigma_{magn.} \sim \frac{E^{1/2}}{(E+\epsilon)(E+\epsilon')}$$

we had: $\sigma_{el} \sim \frac{E^{3/2}}{(E+\varepsilon)^3}$

Must have $g_p + g_n$ with opposite signs

$\sigma_{magn} \ll \sigma_{el}$, since c^3 in denom. for σ_{magn}, only c in denom for σ_{el}.

For ~~large~~ small energy, σ_{mag} becomes larger, compared with σ_{el}. For large E, same energy dependence.



$E + \epsilon = \hbar\omega$.

at 150 kv, $\sigma_{magn} = \sigma_{el}$.

Von Halban: $E \sim 500$ kv

angular dist: $\alpha \sin^2\theta + \beta$.
el magn $\beta \sim .2\alpha$

~~exp:~~ $\dfrac{I_{\theta=0}}{I_{\theta=90}} = \begin{cases} .02 \text{ exp.} \\ .2 \text{ ~~theoretical~~} \end{cases}$

one of the most disconcerting things in nuclear physics

Reverse effect: capture of neutron by proton.

Suppose: 2 effects; one, and its reverse.

$A \rightleftarrows B$.
(neutron) (γ-ray).

Cross-section for any process: $\frac{1}{v}$ (unit density) · Number of final states per unit energy.
incident.
$\sim \left(\frac{p^2 dp}{dE}\right)$ final particle
$\sim \frac{p^2}{v}$ ...

$$\therefore \quad \sigma_{AB} \sim \frac{p^2_B}{v_A v_B}\, \sigma_{\text{interaction}}$$

$$\therefore \quad \sigma_{BA} \sim \frac{p^2_A}{v_A v_B}\, \sigma_{\text{interaction}}.$$

$$\therefore \quad \frac{\sigma_{AB}}{\sigma_{BA}} \sim \frac{p^2_B}{p^2_A} = \frac{\lambda^2_A}{\lambda^2_B}$$ ratio of wave-lengths of incident particles.

$$\therefore \quad \frac{\sigma \text{ ~~phot~~ capture}}{\sigma \text{ photoef}} = \frac{p^2_{\gamma\text{-ray}}}{p^2_{\text{neutron}}} = \frac{(\hbar\,\omega/c)^2}{ME} = \frac{(E+\epsilon)^2}{Mc^2 E}.$$

(c.-of m. coordinates)

$$\therefore \quad \sigma_{\text{magn capture}} \sim \frac{E+\epsilon}{E+\epsilon'} \cdot \frac{1}{\sqrt{E}} \sim \frac{1}{v}$$

Frisch, v. Halban, Koch : $3 \cdot 10^{-25}$ cm^2 for $E = kT$ room temp.

agrees well with theory.

Photon-Scattering coefficient.

4/10/40 Ph~~otomagnetic~~ effect at high energies agrees with ~~theory~~.

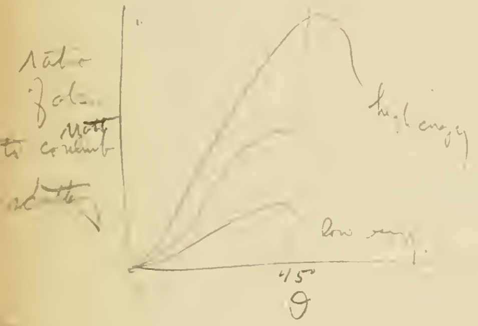
Proton – Proton scattering.

Need new ~~theory~~ ~~to~~ help ~~[illegible]~~ nuclear forces,

White - 1936 - cloud chamber

Tuve + Hafstad '36 '39 - ionization + amplifier 900 kev

Herb + others — 37-39 - 900 - 2400 kev.



Scattering for Coulomb field;

$$\nabla^2\psi + \left(k^2 - \frac{\beta}{r}\right)\psi = 0$$

$$k = \frac{Mv}{\hbar}$$ M reduced mass, v - relative velocity of two ~~particles~~

$$\beta = \frac{2MZ_1Z_2e^2}{\hbar^2}$$

Bohr radius $a_0 = \frac{\hbar^2}{me^2}$;

$$\therefore \beta = 2\frac{M}{m}\frac{Z_1Z_2}{a_0}$$

for P-P scattering, $2M = M_p$, $Z_1 = Z_2 = 1$

$\therefore \beta = \frac{1}{a_p}$, $a_p = \frac{\hbar^2}{M_pe^2} = 3 \cdot$ [illegible]

Want a solution which will be a plane wave at ∞: $\psi = e^{ikz}$

anywhere else, $\psi = e^{ikz}F$ when $F \to 1$ as $z \to \infty$

$$\therefore \nabla^2 F + 2ik\frac{\partial F}{\partial z} - \frac{\beta F}{r} = 0$$

This is solved by $F = F(r-z)$

(because wave equation can be solved in parabolic coordinates)

$$\begin{pmatrix} \xi = r+z \\ \eta = r-z \\ \varphi \end{pmatrix} \qquad \therefore \Psi = \left(1 - \frac{\alpha^2}{ik(r-z)}\right) e^{ikz + i\alpha \log k(r-z)}$$

outgoing spherical wave $\;r \to \;+ \frac{1}{r} e^{i(kr - \alpha \log kr)} \frac{Z_1 Z_2 e^2}{2Mv^2 \sin^2 \frac{\theta}{2}} e^{-i\alpha \log(1-\cos\theta) + i\ldots}$ $(\pi + 2i\eta_0)$

$$\alpha = \frac{2\pi Z_1 Z_2 e^2}{hv} \qquad (\text{if } \alpha \text{ is small, Born approx. better.})$$

Angular distribution given by [illegible]

$$\sigma(\theta) 2\pi \sin\theta \, d\theta = 2\pi \left(\frac{Z_1 Z_2 e^2}{2Mv^2}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}}$$

We must take a f'n antisymmetric in particles;

1) symmetric in spin (triplet)

Antisym space f'n. $\quad \psi(x\,y\,z) = -\psi(-x, -y, -z)$

(relative coords)

$$\therefore \text{take} \quad \Psi = \psi(r, z) - \psi(-r, -z) \qquad F(\theta) \to F(\pi - \theta)$$

$\cos\theta \to -\cos\theta$

2) Antisym in spin - singlet $\qquad \sin \tfrac{1}{2}\theta \to \cos \tfrac{1}{2}\theta$

Sym space f'n $\quad \Psi = \psi(r, z) + \psi(r, -z)$

$\sigma_{trip} \sim |F(\theta) - F(\pi - \theta)|^2$ weight 3

$\sigma_{singlet} \sim |F(\theta) + F(\pi + \theta)|^2$ weight 1

$\therefore \sigma = \frac{3}{4}\sigma_{trip} + \frac{1}{4}\sigma_{singlet}.$

$= |F(\theta)|^2 + |F(\pi - \theta)|^2 - R.P\left(F(\theta) F^*(\pi - \theta)\right)$

$= \frac{1}{\sin^4 \theta/2} + \frac{1}{\cos^4 \theta/2} - \frac{1}{\sin^2 \cos^2} \cos\left(\alpha \log\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\right)$

if α is small, $\cos(\alpha \log(\,)) \sim 1$;

and $\sigma = \sin^{-4}\theta/2 + \cos^{-4}\theta/2 - \sin^{-2}\theta/2 \cos^{-2}\theta/2$

angle θ in center of mass system = 2Θ - lab system;

$\therefore \sigma = \sin^{-4}\Theta + \cos^{-4}\Theta - \sin^{-2}\Theta\cos^{-2}\Theta.$

Particle scattered thru θ
" " " $(\pi - \theta)$

— interference

for $\Theta = 45°$, $\sin\Theta = \cos\Theta = \sqrt{2}/2$

and $\sigma = \frac{1}{2}$ what it would be classically

Deviation from this will indicate other than coulomb forces. At low energies, this is obeyed, at higher, [illegible] nuclear forces enter + cause deviation

4/12/40

Mott: $\left(\frac{Z_1 Z_2 e^2}{2m v^2}\right)^2 \left[\frac{1}{\sin^4\Theta} + \frac{1}{\cos^4\Theta} - \frac{1}{\sin^2\Theta\cos^2\Theta}\cos(\alpha\log\cot\Theta)\right]$

How modified if nuclear interaction exists also?

When we add nuclear ~~potential~~ to coulomb, we must use polar coordinates instead of ~~polar~~ parabolic; or at least we ~~can~~ must add a correction (in polar coord.) ~~to the~~ parabolic expansion; for small r:

$$\Psi(\text{coulom} + \text{nuclear}) = \psi\,\text{coul} + \psi_s(\text{coul} + \text{nucl}) - \psi_s\,\text{coul}.$$

$$\frac{d^2 \psi_s r}{dr^2} + \left(k^2 - \frac{\beta}{r} - \frac{2m}{\hbar^2}V(r)\right)(\psi_s r) = 0.$$

For large r:

$$\psi_l(\text{coulomb}) = \frac{1}{kr}\sin\left(kr - \tfrac{1}{2}l\pi + \eta_l - \alpha\log 2k\right)$$

$$\eta_l = \arg\Gamma(l+1+i\alpha).$$

$$= \arg\left[\Gamma_0 + \frac{1+i\alpha}{\sqrt{1+\alpha^2}} + \frac{2+i\alpha}{\sqrt{4+\alpha^2}} + \cdots + \frac{l+i\alpha}{}\right]$$

$$= \eta_0 + \sum_l \arctan\frac{\alpha}{l}.$$

$$\psi_l(\text{Coul} + \text{Nucl}) - \psi_l(\text{coul}) = \left(e^{2i\delta_l} - 1\right)\frac{1}{2ikr}e^{i(kr - \frac{l\pi}{2} + \eta_l - \alpha\log 2kr)}$$

δ_0 : phase shift $\sim \frac{\sin^2\delta_0}{k^2} \sim \frac{1}{E+\epsilon}$

at high energies, nuclear cross-section greater

at low " , coulomb " "

(since coulomb $\sim \frac{1}{E^2}$)

(also experimentally).

But at small angles, coulomb scatt. will predominate, no matter what energy, because of angular factor. At intermediate angles, interference term comes in; in proton-proton scattering, it helps detect nuclear-scattering of p-waves; Breit: $\delta_1 < .01$ radian, 2400 kv;

Interference also determines sign: $\delta_0 > 0$, ∴ attractive pot.

$\delta_0 > \pi/2$ means real state of He^2, which doesn't exist

∴ $0 < \delta_0 < \pi/2$ (for if $-\pi < \delta_0 < -\pi/2$, as $E \to \infty$, δ_0 would $\to -\pi/2$; but it doesn't).

δ_0 is only unknown in final scattering function; ∴ must be able to adjust it to experimental data, which has always been done. (Bethe ~~thinks~~ this proves Q-M valid for nuclear problems).

Having found δ_0, must determine nuclear potential.

δ_0 varies with energy (in Mott-Prescott, $\tan\delta_0 = \frac{k}{\alpha}$

For rectangular potential, nuclear range $a \approx 2.8 \cdot 10^{-13}$ cm.

For: $\frac{e^{-r/a}}{r/a}$ " " " $a \sim 1.3 \cdot 10^{-13}$.

(Yukawa)

$\lambda = \frac{\hbar}{\mu c}$. mass of meson, from Yukawa potential, $\mu \sim 330\, m$

" " " " cosmic rays, $\mu \sim 200\, m$

For a given range, depth can be calculated accurately; and agrees well with P-N singlet depth. (Bethe thinks this very fundamental).

Many calculations about He^3, H^3, He^4 etc — results negative

4/17/41

4/19/50

$D(\mu)$ = spacing of levels

$\rho(\mu) = \frac{1}{D(\mu)}$ = level density —

$\int \rho(E) e^{-E/T}$ can make this vanish except for $E \sim U$

$$U = \frac{\sum_K E_K e^{-E_K/T}}{\sum_K e^{-E_K/T}} = \frac{\int E \rho(E) e^{-E/T} dE}{\int \rho(E) e^{-E/T} dE}$$

$$e^{-F/T} = \sum e^{-E/T} = \int \rho(E) e^{-E/T} dE .$$

$$-\frac{d e^{-F/T}}{d(\frac{1}{T})} = F e^{-F/T} + \frac{1}{T} e^{-F/T} \frac{dF}{d(\frac{1}{T})} = \int E \rho(E) e^{-E/T} dE$$

$$= e^{-F/T} \left(F - T \frac{dF}{dT} \right)$$

$$\therefore U = F - T\frac{dF}{dT}.$$

$\therefore$ F is like free energy in ~~thermodynamics~~,

then $S = -\frac{dF}{dT}$, $S \to$ entropy.

$$U = F + TS .$$

$$e^{-F/T} = \int \rho(E) e^{-E/T} dE = \frac{\rho(\mu) e^{-\mu/T}}{\lambda(\mu)}$$

$$\therefore \rho(\mu) = \lambda(\mu) e^{\frac{U-F}{T}} = \lambda(\mu) e^{S} .$$

$$\therefore e^{-F/T} = \int \lambda(E) e^{S - E/T} dE = \lambda(\mu) \int e^{S - E/T} dE$$

(since λ varies slowly, + integrand vanishes except for $E \sim$

$$\lambda(\mu) = \frac{e^{-F/T}}{\int e^{S - E/T} dE}$$

$$= \frac{1}{\int e^{\frac{F-E}{T} + S} dE} .$$

but $F = U - TS$

$$\therefore\ \lambda(\mu) = \frac{1}{\int e^{\frac{U - TS^{(\mu)} - E}{T} + S(E)}\, dE}$$

$$= \frac{1}{\int e^{\frac{U-E}{T} + S(E) - S(\mu)}\, dE}$$

but $S(E) - S(\mu) = (E-U)\frac{dS}{dU} + \frac{(E-U)^2}{2}\frac{d^2S}{dU^2}$

but $\frac{dS}{dU} = \frac{1}{T}$;

$$\therefore\ \lambda(\mu) = \frac{1}{\int e^{\frac{(E-U)^2}{2}\frac{d^2S}{dU^2}}\, dE}.$$

$$\frac{d^2S}{dU^2} = \frac{d\frac{1}{T}}{dU} = -\frac{1}{T^2}\frac{dT}{dU}.$$

$$\therefore\ \lambda(\mu) = \frac{1}{\int e^{-\frac{(E-U)^2}{2T^2}\frac{dT}{dU}}\, dE}$$

~~$$= \frac{1}{\sqrt{\pi \frac{dT}{T\,dU}}}$$~~

$$\frac{1}{\sqrt{\pi \frac{2T^2}{\frac{dT}{dU}}}} = \sqrt{\frac{\frac{dT}{dU}}{2\pi T^2}}$$

$$\therefore\ \rho(\mu) = \frac{1}{T}\sqrt{\frac{1}{2\pi}\frac{dT}{dU}}\; e^{S}.$$

$$S(T) = S(0) + \left(\frac{dS}{dT}\right)_0 T + \cdots$$

but $S(0) = 0$;

$\therefore S(T) = aT + \cdots$ non zero.

also since $\frac{dF}{dT} = -S$,

$$F = -\frac{a}{2}T^2 + \cdots$$

$$\therefore U = F + TS = -\frac{a}{2}T^2 + aT^2 = \frac{a}{2}T^2 + \cdots$$

$$\therefore \rho(u) \doteq \frac{1}{T}\sqrt{\frac{1}{2\pi}\cdot\frac{1}{aT}}\; e^{aT}$$

$$\doteq \frac{1}{\sqrt{2\pi a}}\, T^{-3/2} e^{aT}.$$

but $T = \sqrt{\frac{2U}{a}}$

$$\therefore \rho(U) \sim \frac{1}{\sqrt{2\pi a}}\left(\frac{2U}{a}\right)^{-3/4} e^{\sqrt{2Ua}}$$

$$\therefore \rho \sim e^{\sqrt{U}}$$

Can find a by measuring level density;

4/24/40

$$\rho(u) = \frac{1}{T\sqrt{2\pi\frac{\partial U}{\partial T}}}\cdot e^{S}$$

$$= \frac{1}{T\sqrt{2\pi a T}}\, e^{aT}$$

$F = -\frac{a}{2}T^2 \cdots$

$S = -\frac{\partial F}{\partial T} = +aT$

$U = F + TS = \frac{a}{2}T^2$

$\frac{\partial U}{\partial T} = aT$

$$\text{or } \rho(u) = \frac{1}{\left(\frac{2U}{a}\right)^{3/4}\sqrt{2\pi a}}\, e^{\sqrt{2aU}}$$

$$= \frac{a^{1/4}}{2^{3/4}\sqrt{\pi}\, U^{3/4}}\, e^{\sqrt{2aU}}.$$

From γ-ray spectra & neutron ~~[illegible]~~ can say something about level densities:

For ~~[illegible]~~ $\rho \approx \frac{1}{10^5}$ (volts)$^{-1}$

$u_0 = 10^5$

$u_1 = 8 \cdot 10^6 \quad \rho \approx 1 \ (eV)^{-1}$

$$\log \frac{\rho(u_1)}{\rho(u_0)} = \sqrt{2a(u_1 - u_0)} \sim \sqrt{2au_1} \sim 10^{3} \sqrt{16a}$$

$$\log 10^5 \sim 4 \cdot 10^3 \sqrt{a}$$

$$121 \sim 16 \cdot 10^6 a$$

$$a = \frac{121}{16 \cdot 10^6} \sim 7 \cdot 10^{-6}.$$

$$T_1 = \sqrt{\frac{2u_1}{a}} \sim \sqrt{\frac{16 \cdot 10^6}{7 \cdot 10^{-6}}} = \frac{4}{3} \cdot 10^6$$

(fairly heavy nucleus) ~~T almost independent of a~~; change ρ by factor of 10 — T changes by about 40%.

---

a —

↑↓

b —

$g_a P_{ab} = g_b P_{ba}$

g: statistical wt. of state; P_{ab}: probability of process $a \to b$; P_{ba}: $b \to a$

a = 1st state: excited comp. ~~nucleus~~

b = 2nd state: nucleus in ground state, and free particle (neutron)

$$P_{ab} = \gamma_n / \hbar \qquad g_a = \rho(E)\, dE$$

P_{ba} – in terms of cross section

suppose particle density in box Ω is $N(E)\,dE$

then ~~P_{ba}~~ = no. of processes $= N(E)\,dE \ P_{ba}$; or

or ~~no. of~~ ~~cross-section~~ =

prob. of process = density · velocity · cross-section.

$$2 = \frac{N}{\Omega} v \sigma$$

5/1/40

$$\frac{\rho(u_1)}{\rho(u_2)} = e^{\sqrt{2cu}} = 10^5 \quad \text{at 8 m.e.V}$$

$$\therefore e^{\sqrt{16c}} = 10^5 \qquad \therefore c = 9 \quad \text{approx.}$$

$$\frac{\rho(16)}{\rho(8)} = e^{\sqrt{18(16-8)}} = e^{3.4(\sqrt{2}-1)} = e^{12.4} \sim 100 .$$

u	T	$\frac{1}{\rho}$
0	0	10^{-1}
8	1.3	10^{-6}
16	1.8	10^{-8}

all in m.e.v.

$$T = \sqrt{\frac{2u}{c}}$$

Principle of detailed ~~balancing~~ requires ~~that~~:

C - compound nucleus
A - resulting nucleus
P. particle emitted

$$g_c W^c_{AP} = g_P g_A W^{AP}_c$$

g's - statistical wts.

W's - probabilities of process wt

$$W^c_{AP} = \frac{\gamma^c_{AP}}{\hbar}$$

$$W^{AP}_c = \frac{v\, \sigma^{AP}_c}{\Omega}$$

$$g_c = \rho(u)\, du$$

$$g_A = 1$$

$$g_P = \frac{\Omega\, k^2 dk}{2\pi^2} \qquad \left(\frac{\Omega 4\pi p^2 dp}{h^3} \;;\; p = \hbar k \;;\; \therefore = \frac{\Omega\, 4\pi k^2 dk}{8\pi^3} = \right.$$

$$\therefore \rho(\mathcal{U}) \frac{d\mathcal{U}}{\hbar} \gamma^{c}_{AP} = \frac{\Omega k^2 dk}{2\pi^2} \cdot \frac{v\,\sigma_c^{AP}}{\Omega}$$

$$= \frac{v\,\sigma_c^{AP} k^2 dk}{2\pi^2}$$

$$\therefore \sigma_c^{AP} = \frac{2\pi^2 \gamma^c_{AP}\, \rho(\mathcal{U})}{v k^2 \hbar} \frac{d\mathcal{U}}{dk}.$$

$$\mathcal{U} = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\frac{d\mathcal{U}}{dk} = \frac{2\hbar^2 k}{2m}$$

$$\therefore \sigma_c^{AP} = \frac{2\pi^2 \gamma^c_{AP}\, \rho(\mathcal{U})}{v k^2 \hbar} \cdot \frac{\hbar^2 k}{m}$$

$$= \frac{2\pi^2 \hbar\, \gamma^c_{AP}\, \rho(\mathcal{U})}{mv\, k}$$

$mv = p = \hbar k$

~~but $\frac{\hbar}{mv} = p$~~

$$\therefore \sigma_c^{AP} = \frac{2\pi^2}{k^2} \rho(\mathcal{U})\, \gamma^c_{AP}$$

With spin:

$g_A = 2i+1$ (i - spin of nucleus)

$g_{c_J} = \rho_J(\mathcal{U})\,d\mathcal{U}$ - for ang. mom. J ; $\rho_J(\mathcal{U}) = \frac{2J+1}{d_J}$

d_J = dist. betw. levels

$g_p = (2s+1) \frac{\Omega k^2 dk}{2\pi^2}$

Then $\sigma_{c_J}^{AP\ell} = \frac{2\pi^2}{k^2} \frac{(2J+1)}{(2i+1)(2s+1)} \frac{\gamma^{c_J}_{AP\ell}}{d_J}$

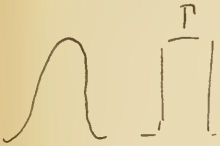
Now maximum cross section for a given ℓ is: $\sigma^{(\ell)}_{max} = \frac{\pi}{k^2}(2\ell+1)$

$\therefore \frac{\gamma}{d} \sim 1$ small wave length: $\sum_0^{kR} \sigma^{(\ell)}_{max} = \frac{\pi}{k^2} \sum_0^{kR} (2\ell+1) = \pi R^2$

5/8/40

$$\int \sigma^{AP}_{CJ}\, dE = \frac{2\pi^2}{k^2}\,\frac{2J+1}{(2i+1)(2s+1)}\,\gamma^{AP}_{CJ}$$

$$(\sigma^{AP}_{CJ})_{Max} \sim \frac{2\pi^2}{k^2}\,\frac{\gamma^{AP}_{CJ}}{\Gamma_{CJ}}.$$



$$(\sigma^{AP \to CJ}_{\swarrow BQ})_{max} = (\sigma^{AP}_{CJ})_{Max}\,\frac{\gamma^{CJ}_{BQ}}{\Gamma_{CJ}} = \frac{2\pi^2}{k_P^2}\,\frac{\gamma^{CJ}_{AP}\,\gamma^{CJ}_{BQ}}{(\Gamma_{CJ})^2}.$$

Bethe & Placzek: ~~treat~~ comp. nucleus like photon in Dirac radiation theory — ~~introduce~~ ~~interaction~~ w. particles

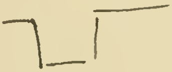
Kapur, Peierls; ~~investigate~~ changes in boundary conditions —

First: (C) χ_C; [scribbled out] Then (B) $\psi_B \cdot \varphi_P^{OP}$.

might require $\chi_C = 0$ for $r > r_0$;

but this would need infinite ~~potential~~

so ~~the~~ ~~finite~~ ~~potential~~:



$$\frac{d^2\varphi}{dr^2} + \frac{2m}{\hbar^2}(E-V)\varphi = 0 \qquad r < r_0$$

$$\frac{d^2\varphi}{dr^2} + k^2\varphi = 0 \qquad r \geq 0 \qquad k^2 = \frac{2mE}{\hbar^2}$$

$$\varphi = I\,\frac{\sin kr}{k} + S e^{ikr} \qquad \varphi' = I\cos kr + ikSe^{ikr}$$

$\therefore ik\varphi - \varphi' = iI\sin\ldots$
$- I\cos k\ldots$
$= -I e^{-ik\ldots}$

If system can only decay: only outgoing waves;

no incoming waves if: $\frac{d\varphi}{dr} = ik\varphi$; (then $I=0$).

Can satisfy this & 1st eq'n only for certain values of k — (complex)

if $W_n = E_n + i\frac{\gamma_n}{2}$.

$$e^{\frac{iW_nt}{\hbar}} = e^{\frac{iE_nt}{\hbar}}\, e^{-\frac{\gamma_n t}{\hbar}}$$

∴ complex eigen value means exponential decay of nucleus;

5/15/40

$\gamma_n =$

5/15/40 $\gamma_n = \frac{\hbar^2}{2m} |\varphi_n(r_0)|^2 k_n$ $W_n = E_n - \frac{i\gamma_n}{2}$

$k_n = \frac{\sqrt{2mE_n}}{\hbar}$

I incident wave present: k – wave vector of outgoing wave –

boundary cond.: $I e^{-ikr} = \left(\frac{d\varphi}{dr}\right)_{r_0} - ik\varphi(r_0)$

solutions φ_n (of eqn without incident wave)

build up $\varphi = \sum a_n \varphi_n + \chi$, which satisfies boundary condition – (incident wave)

then $\left(\frac{d\chi}{dr}\right)_{r_0} - ik\chi(r_0) = I e^{-ikr_0}$

$\left(\varphi = I \frac{\sin kr}{kr} + S e^{ik\ldots}\right.$

can determine a_n's ;

Kapur, Peierls

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$S = \cos kr_0 \, \varphi(r_0) - \frac{1}{k}\sin(kr_0)\left(\frac{d\varphi}{dr}\right)_{r_0}$

$\sigma_B^A = \frac{4\pi |S|^2}{|I|^2} = \frac{\pi}{k^2}\left|\sum_n \frac{u_A^n u_B^{n*}}{(W_n - E) N_n}\right|^2$ E: energy incident p…

$\gamma_A^n = |u_A^n|^2$

$N_n = \int_{\text{nucleus}} \varphi_n^2 \, d\tau$

$= 1$ almost

When level density is great, can separate nuclear quantities into those varying slowly (order of nucl… Mev binding energies) & those " fast (order of level separation volts). Here only slow variations important ; if $E < |W_n|$ $\left(E \ll E_n,\ E \ll \gamma_n\right.$ $|W_n - E| \gg \Delta E$

σ varies only as $1/k^2$

reaction probability $= P = j\sigma = \rho P$) but $j = \rho v$, $\therefore \sigma = \frac{P}{v}$

5/17/40

$$\sigma^A_B = \frac{\pi}{k^2}\left|\sum_n \frac{u_A u_B}{(W_n - E)}\right| \qquad |u^n_A|^2 = \gamma^n_A$$

$$W_n = E_n - i\frac{\gamma_n}{2}$$

If near resonance, so that only one term ~~contributes~~ to cross-section:

$$\sigma^A_B\rangle^n = \frac{\pi}{k^2}\frac{\gamma^n_A \gamma^n_B}{(E_n - E)^2 + \left(\frac{\gamma_n}{2}\right)^2}$$

or, if $x = \dfrac{E - E_n}{\gamma_n/2}$

$$\sigma^A_B\rangle^n = \frac{4\pi}{k^2}\frac{\gamma^n_A\gamma^n_B}{\gamma_n^2}\frac{1}{1+x^2} = \frac{\sigma_0{}^{A,n}_{B}}{1+x^2}$$

$$\int \sigma^{A,n}_{B}\, dE = \gamma_n/2 \int \frac{\sigma_0{}^{A,n}_{B}}{1+x^2}\,dx = \sigma_0{}^{A,n}_{B}\cdot\frac{\gamma_n}{2}\cdot\pi$$

$$\therefore = \frac{2\pi^2}{k^2}\frac{\gamma^n_A\gamma^n_B}{\gamma_n} \text{ ~~[illegible]~~ };$$

comparing with 5/8/40, we see that, if there are angular momenta, ~~statistical~~ factor $\dfrac{(2J+1)}{(2i+1)(2s+1)}$ is added:

$$\therefore \sigma^{A,n}_{B} = \frac{4\pi}{k^2}\frac{(2J+1)}{(2i+1)(2s+1)}\frac{\gamma^n_A\gamma^n_B}{(\gamma^n)^2}\frac{1}{1+x^2}$$

$$\sigma_0 = \sim \frac{\Gamma^n_A\Gamma^n_B}{(\Gamma^n)^2}$$

$$\Gamma^n_A = \frac{k_n}{k_0}\gamma^n_A \; : \quad \sigma^{A,n}_{B} = \sigma_0{}^{A,n}_{B}\cdot\frac{1}{1+x^2}\cdot\frac{k_0}{k}$$

because for slow particles, γ^n_B is practically indep. of k —

$$J = |i-s| \ldots i+s$$

for a neutron: $J = i \pm \frac{1}{2}$ $\quad\therefore \dfrac{2J+1}{(2i+1)\cdot 2} = \dfrac{2i\pm 1+1}{(2i+1)2} = \dfrac{1}{2} \pm \dfrac{1}{2(2i+1)}$

β low neutrons:

~~Abstract~~

Total cross section : $\sigma_{max} = \frac{2\pi}{k^2}\left\{1 \pm \frac{1}{2i+1}\right\}\frac{\Gamma_{neutron}}{\Gamma_r}$ $\left[\frac{\Gamma_1}{\Gamma_1+\Gamma_{ne}} \sim 1\right]$

Γ_{ne} - neutron width. Γ_n ~ Γ_1 radiation width ~ (total due to point...

$$\sigma_{scattering} = \sigma_{max}\left(\frac{\Gamma_{ne}}{\Gamma_1}\right) = \frac{2\pi}{k^2}\left(1 \pm \frac{1}{2i+1}\right)\left(\frac{\Gamma_{ne}}{\Gamma_r}\right)^2$$

5/22/40

$j = \rho v$ $\qquad j\sigma = \rho p$

$\therefore \sigma = \frac{p}{v}$

in system moving with velocity v':

$$\sigma(v) = \frac{v'}{v}\sigma(v')$$

---

v – vel. in observer's system.

v' – vel. in system in which nucleus at rest

u – vel. of nucleus.

then $v = v' + u_x$

$v' = v - u_x$

$E' - E = mvu_x = \sqrt{2mE}\; u_x$

If Maxwell ~~distribution~~ : $w(u_x)du_x = \left(\frac{M}{2\pi kT}\right)^{1/2} e^{-\frac{Mu_x^2}{2kT}} du_x$

$$\sigma(E) = \int \sigma(E')\, W(E')\, dE'$$

We had: $\sigma(E') = \dfrac{\sigma_0}{1+\left(\frac{E'-E_0}{\Gamma/2}\right)^2}$ put: $\dfrac{E'-E_0}{\Gamma/2} = x'$

$\dfrac{E-E_0}{\Gamma/2} = x$

$\therefore \sigma(E) = \sigma_0 \int \dfrac{}{1+x'^2}$

$w(E')\,dE' = \left(\dfrac{M}{2\pi kT}\right)^{1/2} \dfrac{1}{\sqrt{2mE}} e^{-\frac{1}{2kT}\dots}$

$$\therefore \sigma(E) = \frac{\sigma_0 \quad \Gamma/2}{\sqrt{4\pi kTE\frac{m}{M}}} \int \frac{e^{-\frac{M(x-x')^2\,\Gamma/2}{4kTmE_0}}\,dx'}{1+x'^2}.$$

call: $4\sqrt{kTE_0\frac{m}{M}} = \Delta$ $x'-x = \sqrt{2mE_0}$ $\Gamma/2$

$\Gamma/\Delta = \xi.$

$$\sigma(E) = \sigma_0(\pi)^{-1/2}\xi \int \frac{e^{-\xi^2(x-x')^2}}{1+x'^2}\,dx'$$

if ξ is large, almost whole integral comes from near $x'=x$;

$$\therefore \sigma = \frac{\sigma_0\,\xi}{\sqrt{\pi}(1+x^2)} \cdot \frac{\sqrt{\pi}}{\xi} = \frac{\sigma_0}{1+x^2}.$$

if ξ is small, $\sigma = \dfrac{\sigma_0}{\sqrt{\pi}}\xi\, e^{-\xi^2x^2} \displaystyle\int \frac{dx'}{1+x'^2}$

$$= \sigma_0\sqrt{\pi}\,\xi\, e^{-\xi^2x^2}$$

$\xi x = \dfrac{E-E_0}{\Delta/2}$

$$= \sigma_0\sqrt{\pi}\,\frac{\Gamma}{\Delta}\, e^{-\left(\frac{E-E_0}{\Delta/2}\right)^2}$$

really doppler effect – Δ – doppler width.

$\Delta = 4\sqrt{hT\frac{m}{M}E_0} \sim \frac{4}{\sqrt{40}}\frac{1}{\sqrt{100}}\cdot\sqrt{E} \sim \frac{1}{15}\sqrt{E}$

This was all for gases —

for solids : same results, if $W >> k\Theta$ - only —

substitute mean energy, degree of freedom for ~~temperature~~ -

---

5/24/40

$\xi >> 1 : \sigma = \frac{\sigma_0}{1+x^2}$; $\int \sigma\, dE = \pi \frac{\Gamma}{2}\sigma_0$

$\xi << 1 \quad \sigma = \sigma_0 \xi \sqrt{\pi}\, e^{-\xi^2 x^2}$ — Doppler

---

neutron, energy E_0 passes into water or paraffin; what will be energy distribution? -

Q neutrons per sec.

(logarithmic convenient)

$$\rho(E)\frac{dE}{E} = Q \frac{l(v)}{v} K(x)\frac{dx}{x} \qquad x = \frac{E_0}{E}.$$

$l(v)$: mean free path; $\frac{l(v)}{v}$. lifetime

($K(x)\,dx$ represents av. no. of collisions in dx)

$k(x,x')\frac{dx}{x}$: energy distribution after collision -

$\therefore$ ~~total~~ no. going from x to x' is : $\int_1^x k(x,x')K(x')\frac{dx\,dx'}{x\,x'}$

$$\therefore K(x) = k(x,1) + \int_1^x k(x,x')K(x')\frac{dx'}{x'}$$

For neutron - proton collision, $k(x,x')\frac{dx}{x} = \frac{dE}{E'} = -\frac{x'}{x}\frac{dx}{x}$

$\therefore K(x) = \frac{1}{x} + \int_1^x \frac{x'}{x}K(x')\frac{dx'}{x'}$; $K(x) = \frac{1}{x}\left[1 + \int_1^x K(x')\,dx'\right]$

$\therefore K(x) = 1$

$$\therefore \quad \rho(E)\frac{dE}{E} = \frac{Q\,\ell(v)}{v}\frac{dE}{E} = 2Q\,\ell(v)\frac{dv}{v^2}$$

$$B^{10} + n = Li^7 + He^4$$

5/2?/?0

β – disintegration

(No essential differen. betw. [illegible] disintegr.)

We found: if $Z^A > (Z+1)^A$, get $Z^A \to (Z+1)^A + \beta^-$
(atoms with [illegible])

if $(Z+1)^A > Z^A + 2m$: get $(Z+1)^A \to Z^A + \beta^+$

if $Z^A < (Z+1)^A < Z^A + 2m$

can have : $(Z+1)^A + \varepsilon^- \to Z^A$
(K-absorption [illegible]) [illegible]

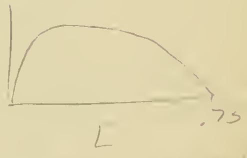
Introduce new [illegible] to [illegible]

$$P \rightleftarrows N + \beta^+$$

conflict in conservation of ~~statistics~~ + spin

$[N + \beta^+]$ must have Fermi ~~stat.~~; but since [illegible]
[illegible] must have B. E. stat; [illegible], P
F. D. stat., since must have [illegible]

have spin ½, as do N + β [illegible]

... ... particle having Fermi stat.,)
... ... charge, ... — called
neutrino
$$P \rightleftarrows N + e^+ + \nu$$
... ... positron ... ; ... Fermi theory
... ... creation negative electron — similarly,
... ... neutrino ... as product of anti-neutrino;
i.e. $N \rightleftarrows P + e^- + \nu'$
$N - (P + e) = .75$ Mev.
The possible ... —
if ... ... .75 Mev ...
... ...
... ... energy ... of e^- particle;
... ... —

... ... the neutrino!
... ... this, was that ... of
... ... its significance ; conservation
... energy would ... ... true in any
... ... The average energy of ... would
... ... content ... ... final ... ,
... difference in energy ... above ...
... ... cut-off —
... ... and
... ... a ... limit.

Experimentally, ... neutrino ... required.

Lyman California N^{13} ...

... $C^{13} + H \rightarrow N^{13} + n$

This reaction has a threshold

$3.20 \pm .03$ Mev

$Q = \frac{13}{14} \cdot 3.20 = 2.97 \pm .03$

Assuming $n - H = .76$ Mev, (Bethe ...)

then $N^{13} - C^{13} = 2.21 \pm .06$ Mev

-1.02 ($= 2mc^2$)

Max kinetic energy: $1.19 \pm .06$ Mev.

Max energy of N^{13} ... $= 1.198 \pm .006$ Mev.

Whereas average energy ... $\sim .5$

hence neutrino theory is correct —

Mass of neutrino ... $\pm .06$ Mev

Assume a Hamiltonian: $H = g\, Q\, \varphi^{*} \psi \ ... \ \varphi \psi \varphi^{*}$

Q converts neutron to proton ...

φ — initial state of neutron

ψ — final state of neutron

φ ... neutrino state

$H = g \psi \varphi$; Matrix element $= \int \Psi_i^{*} H' \Psi_f \, d\tau$

ψ_n : [illegible] function

ψ_p - [illegible]

$H_{if} = g \int d\tau \; u_n^* \, \psi_e^* \, u_p \, \psi_\nu$

[illegible] $|M|^2$ [illegible] =

[illegible] $\psi_e = a \, e^{i k \cdot r}$

$\psi_\nu = a_\nu \, e^{i k \cdot r}$

[illegible] $\frac{4\pi p^2 dp}{(2\pi\hbar)^3}$

[illegible] of M [illegible] energy, g [illegible]

[illegible] energy [illegible].

$$\frac{2\pi}{\hbar} g^2 \left| \int d\tau \; u_n^* \, u_p \, a_{\text{e}}^* e^{-i k_e \cdot r} \, a_\nu e^{i k_\nu \cdot r} \right|^2$$

$$\cdot \frac{4\pi p_e^2 \, dp_e \cdot 4\pi p_\nu^2 \, dp_\nu}{(2\pi\hbar)^6 \, d(E_e + E_\nu)}$$

keep E_n const., then $\longrightarrow \frac{1}{4\pi^4} p_e^2 \frac{dp_e}{dE_e} \cdot \frac{1}{c^3} E_\nu^2 \, dE$

$$\longrightarrow \frac{1}{4\pi^4 c^6} \sqrt{E_e^2 - m^2 c^4} \; E_e E_\nu^2 \, dE$$

(probability of emission of neutrino with [illegible] between E_ν and $E_\nu + dE_\nu$)

E_e [illegible] $E_{max} - E_e$ [illegible]

$$= \frac{1}{4\pi c^5 \hbar^7} \sqrt{E_e^2 - m^2 c^4}\, E_e (E_m - E_e)^2 \, dE_e$$

if $E_n \sim mc^2$, $\quad h_n \sim \frac{mc}{\hbar}$ and [illegible]

[illegible] if E_n [illegible] (most energetic [illegible])

take $f_m = 4\alpha = 1$

then we [illegible] $|O_n^{+} O_{el}|^2$ [illegible] $\int \psi^{*} u_n^{*} u_p$

$\hookrightarrow$ averaged over space [illegible] gives 1

$\bar{G} \leq 1$ [illegible]

for [illegible] $\bar{G} \ll 1$ [illegible]

we have $\quad \alpha(\varepsilon) = \frac{2\pi}{\hbar} \frac{g^2}{4\pi^4 c^6 \hbar^6} \cdot m^5 c^{10} \sqrt{\varepsilon^2 - 1}\, \varepsilon(\varepsilon_0 - \varepsilon)^2 d\varepsilon$

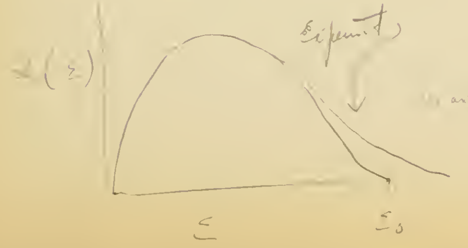
$\varepsilon = \frac{E_e}{mc^2}$

$$= \frac{g^2 m^5}{2\pi^3 c^4 \hbar^7} \sqrt{\varepsilon^2 - 1}\, \varepsilon(\varepsilon_0 - \varepsilon)^2 d\varepsilon$$

$\varepsilon_0 = \frac{E_{max}}{mc^2}$

total [illegible] $\lambda = \int \alpha(\varepsilon)\, d\varepsilon$

$$\sim \int \sqrt{\varepsilon^2 - 1}\, \varepsilon(\varepsilon_0 - \varepsilon)^2 d\varepsilon$$



[illegible]

[illegible] function of [illegible] ; calculated [illegible] $\frac{E_0}{[illegible]}$

[illegible] $|M|$, [illegible] $\alpha(\varepsilon)$ has calculated $(\varepsilon_0 - \varepsilon)^2$ which

[illegible] for [illegible] probability not [illegible], [illegible] agreement [illegible]

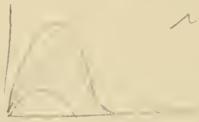
exp. Later, [illegible] that experimental [illegible] was

[illegible] ; + exp. [illegible] quite agree with either theory;

Peierls + [illegible] than [illegible], [illegible] exp. [illegible] curves are

sum of [illegible] : worked [illegible] [illegible] [illegible] ;

)



[illegible] [illegible] of [illegible] [illegible] [illegible]

[illegible] Fermi [illegible] I^{111} P^{32} [illegible] [illegible]

[illegible] well.

Total probability $\alpha = \int \alpha(\varepsilon)\, d\varepsilon$

$$= \frac{1}{?} |\mathcal{G}|^2 \int_1^{\varepsilon_0} d\varepsilon \, \sqrt{\varepsilon^2 - 1} \, \varepsilon (\varepsilon_0 - \varepsilon)^2$$

[illegible] [illegible] $\frac{t}{|\mathcal{G}|^2}$; [illegible] [illegible] T_0 $T_0 \sim 1$ hour

[illegible] [illegible] [illegible] ; [illegible] [illegible]

[illegible] [illegible] nuclei H^3 [illegible] Li [illegible] Be [illegible]

$Z_p, Z{-}1_n \to Z_n, Z{-}1_p$.

Natural [illegible] [illegible] $|\mathcal{G}| \sim \frac{1}{100}$

Selection rules $\Delta J = 0$, for $G = 0$ if neutral & final nuclei have different energy ...

Gamow-Teller: $G = \int d\tau\, u_n^{*} \sigma u_p$ $\quad \Delta J = \pm 1, 0$

Evidence favors G-T selection rule with the Fermi

$\Delta J > 1$, then expand of [illegible]; but in most ... [illegible] must have electron ... for ... accounted —

forbidden transitions ... [illegible] allowed —

Fermi theory:

Allowed	$\Delta J = 0$	1
Forbidden	$\Delta J = 1$	10^{-2}
	$\Delta J = 2$	10^{-5}

... 1st for ... [illegible]

Dipole moment: $\int u_a^* \psi_a^* \ldots (\sum e_i \ldots) \ldots$

Case 1, $n_a = n_b$

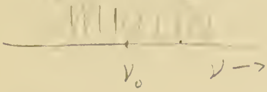
get $\Delta j = \pm 1$

$j \to j-1$ energy diff. $= \frac{\hbar^2}{2I} \ldots$

Case 2: $n_a = n_b$

get vibration band

$\Delta j = 1$ $\Delta j = -1$

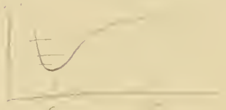


$$\left\{\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}\right) - \frac{1}{\sin^2\theta}\left(\ldots\right)\right\}\mathcal{Q} = \ldots\mathcal{Q}$$

rotational states –

$$\left\{-\frac{\hbar^2}{2M}\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{d}{dR}\right) + \frac{\hbar^2}{2M}\frac{\ldots}{R^2} + W_i(r) + \bar{U}_e(r)\right\} P = W P$$

vibration of states –



[illegible] potential energy

$$\left\{-\frac{\hbar^2}{2M}\frac{1}{R^2}\frac{d}{dR}R^2\frac{d}{dR} + \ldots\right\} P = \ldots$$

and $W = W_e(R_0) + \bar{U}_e(R_0) + \frac{\hbar^2}{2M}\frac{\ldots}{R_0^2} + W_v$

Simplest case: [illegible] rotating molecules. take $\lambda = 0$, $S = 0$, Σ

then $\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\sin\theta\frac{d}{d\theta} - \frac{m^2}{\sin^2\theta}\right]\mathcal{Q} = 0\,\mathcal{Q}$ $\quad \mathcal{Q} = j(j+1)$

rotational energy: $\frac{\hbar^2}{2M R_0^2}\, j(j+1)$

$$\left\{-\frac{\hbar^2}{2M}\frac{1}{\ldots} - \frac{\hbar^2}{\ldots}j(j+1)\frac{1}{R_0^2} \ldots\right\} + \ldots$$

expand [illegible] about R_0 [illegible] $(R - R_0)^2$

[illegible]

[illegible]

Of ... the rotation effect ... [illegible]

a ... ; ([illegible])

[illegible] $\underline{O} = L$ ... ,

I use ... equation for ...)

$$\psi = e^{im\varphi} \mathcal{B}(\theta) \, F \ldots u \ldots$$

$$-\frac{\hbar^2}{2M}\Big[u \, \Delta\big(e^{im\varphi} \mathcal{B} P\big) + e^{im\varphi} \mathcal{B} \Big\{ \frac{2}{r} \ldots + \ldots + P \ldots$$

$$+ \frac{P}{R^2}\Big[-i \,(\cot\theta + 2 \tfrac{\partial}{\partial\theta}) L_y' + \frac{2i}{\sin\theta}(L_x' - L_3' \ldots) \ldots$$

$$-L_1'^2 - L_y'^2 - L_3'^2 \cot^2\theta + (L_x' L_y' + L_3' \ldots)\cot\theta \Big] u \Big\}$$

$$+ e^{im\varphi} \mathcal{B} P \Big\{ \frac{Z_1 Z_2 e^2}{R} - \sum e^2\Big(\frac{Z_1}{r_{1\tau}} + \frac{Z_2}{r_{2\tau}}\Big) + \sum\sum \frac{e^2}{r_{\tau\mu}}$$

$$-\frac{\hbar^2}{2m}\sum \Delta \ldots + \ldots \Big\} u = W e^{im\varphi} \mathcal{B} P u .$$

$$\frac{e\hbar^2}{4m^2c^2}\sum_{\tau} \Big[\ldots + 2i(\ldots) \ldots \Big]$$

Take first bracket & find its ... [illegible]

$$\Big\{ \frac{Z_1 Z_2 e^2}{R} \ldots - [\ \ldots\ ] \Big\} u = W_e(R) u .$$

... $L_3' u = \Lambda' u$ quantum no's $\Lambda = |\Lambda'|$ $\quad \Lambda = 0 \ 1 \ 2 \ 3$

$O_2' u = \Omega' u$ $\quad \Sigma \ \Pi \ \Delta \ \Phi$

Treat second bracket (... + ...) ...

... $W_e(R) = -\frac{\hbar^2}{M}\Big[\ldots \Big]$ ...

(1)

$$-\frac{\hbar^2}{M'}\Delta + \frac{Z_1 Z_2 e^2}{R} - \sum e^2\left(\frac{Z_1}{r_{1\tau}} + \frac{Z_2}{r_{2\tau}}\right) + \sum_{\tau<\mu}^{N} \frac{e^2}{r_{\tau\mu}}$$

$$-\frac{\hbar^2}{2m}\sum \Delta_\tau + \frac{e\hbar^2}{4m^2c^2} \sum\left(E_\tau\cdot\nabla_\tau + 2i(E_\tau\times\nabla_\tau)\cdot S_\tau\right)\psi = W\psi$$

equation

$R_1 = \frac{M_1}{M_1+M_2}R$

$R_2 = \frac{M_2}{M_1+M_2}R$

$R\ \Theta\ \Phi$

$X = R\sin\Theta\cos\Phi$

$Y = R\sin\Theta\sin\Phi$

$Z = R\cos\Theta$

$x_\tau' = x_\tau\cos\Phi\cos\Theta + y_\tau\sin\Phi\cos\Theta - z_\tau\sin\Theta$

$y_\tau' = -x_\tau\sin\Phi + y_\tau\cos\Phi$

$z_\tau' = x_\tau\cos\Phi\sin\Theta + y_\tau\sin\Phi\sin\Theta + z_\tau\cos\Theta$

$S_\alpha' = S_\alpha\, e^{i\Phi/2}\cos\frac{\Theta}{2} + S_\beta\, e^{i\Phi/2}\sin\frac{\Theta}{2}$

$$-\frac{\hbar^2}{M}\left\{\Delta + \ldots \left(L_x{}^2 + L_y{}^2 - L_3{}^2\right)\cot\Theta + \left(L_x L_3 + L_3 L_x\right)\cot\Theta\right]$$

$$+\frac{Z_1 Z_2 e^2}{R} - \sum e^2\left(\frac{Z_1}{r_{1\tau}} + \frac{Z_2}{r_{2\tau}}\right) + \sum\sum\frac{e^2}{r_{\tau\mu}} - \frac{\hbar^2}{2m}\sum\Delta_\tau$$

$$+\frac{e\hbar^2}{4m^2c^2}\sum\left[E_\tau\cdot\nabla_\tau + 2i(E_\tau\times\nabla_\tau)\cdot S_\tau\right]\psi = W\psi$$

or $H_I = \frac{e\hbar}{mc} \sum_{T} \frac{1}{r_T^3}(\ldots$ [illegible]

F commutes, ∴ H_I separate [illegible]

$W_{\ldots} = A[\ldots]$ [illegible]

$\Delta W = 2A(F+1)$

can find I & F [illegible]

Sommerfeld, [illegible]

Band Spectra (Herzberg, [illegible]

(Mulliken [illegible])

5/30/41

$$H = -\hbar^2 \sum_j \frac{1}{2M_j}\Delta_j + \sum_{j<k}\sum \frac{Z_j Z_k e^2}{R_{jk}} - \sum_j \sum_T \frac{Z_j e^2}{r_{jT}}$$

$$+ \sum\sum \frac{e^2}{r_{T\mu}} - \frac{\hbar^2}{2m}\sum_T \Delta_T + \frac{e\hbar^2}{4m^2c^2}\sum_T \left[\mathcal{E}_T \cdot \nabla_T + 2i(\mathcal{E}_T \times \nabla_T)\cdot s_T\right]$$

$$\mathcal{E}_T = \nabla_T \sum_j \frac{Z_j e^2}{r_{jT}} - \nabla_T \sum_{\mu \neq T} \frac{e^2}{r_{T\mu}}$$

General properties of H:

1. J commutes with H

[illegible]

∴ Quantum no. J [illegible]

2. Positive & Negative states:

[illegible]

[illegible] (discrete energy)

[illegible] (continuum)

$H_1 = \sum\sum \frac{e^2}{r}$ [illegible]

$H_{[illegible]} = \int [illegible]$

[illegible]

Hyperfine structure:

Two types — isotope structure [illegible]
nuclear spin [illegible]

Nuclear ang. mom. I.

magnetic interaction [illegible] (Field degeneracy) [illegible]

[illegible]

$J + I = F$, total ang. mom.

[illegible] F, M_F

If $J > I$ [illegible]

add interaction

Nuclear mag. mom. μI

Field of magnetic moment μI: $A = \frac{\mu}{r^2} I \times r_1$

$H = \text{curl}\, A = \frac{\mu}{r^3}\left[3(I\cdot r_1) r_1 - I\right]$

r_1: unit vector [illegible]

Add to Hamiltonian:

$-\frac{e\hbar}{mc} A\cdot\nabla - \frac{e\hbar}{mc} M\cdot S$ [illegible]

Ershun [illegible] 11 electr. 1802 ... 200 ...

Calculate Hartree field, then

take one K electron out – [illegible]

calculate field

Auger Transition –

Pseudo states: if a minor term in Hamiltonian is omitted, discrete states occur with same energy as [illegible]

Apparent transition occurs when [illegible]

[illegible] discrete [illegible]

$H = H_0 + H_1 \qquad H_0 u_m = W_m^0 u_m \qquad H_0 u_j = W_j^0 u_j \qquad \frac{\partial H}{\partial t} = \ldots$

$\psi = \sum_n a_n u_n e^{-i\zeta W_n^0 t} + \int a_j u_j e^{-i\zeta W_j^0 t} dj$

then $\dot{a}_j = -i\zeta \sum_k a_k H_{jk} e^{i\zeta(W_j^0 - W_k^0)t}$ [illegible] $H_{jk} = \int u_j^* \ldots$

at $t = 0$ set $\psi = u_n$ then [illegible] $\dot{a}_j = -i \ldots e^{i\zeta(W_j - W_n)t}$

$a_j = -H_{jn} \frac{e^{i\zeta(W_j^0 - W_n^0)t} - 1}{W_j - W_n}$

$|a_j|^2 = |H_{jn}|^2 \ldots$

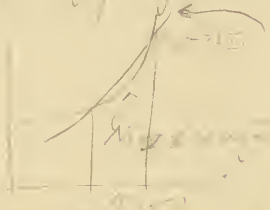
$\int |a_j|^2 dj = \frac{4\pi^2}{h^2} |H_{jn}|^2 t$

Auger Trans. [illegible] $= \frac{4\pi^2}{h} |H_{jn}|^2$

$H_{jn} = \int u_j^* H_1 u_n \ldots$

5/3/48 ... vorticity depth

... energy eq ... M_z ...



Reading ... 37, ...

... $KL \to L$ ...

$(KL)_z = K_z + L_{z+1}$

L'_z

$- (\ldots) \to 0$

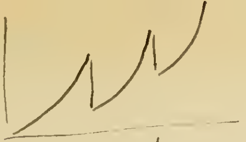
or $\ldots - (LM)_z = K'_z - L_z + (L_{z+1} - M_{z+1})$

$= (K_z - M_z) + (L_{z+1} - L_z) - (M_{z+1} - M_z)$

LL

5 lines — ...

Absorption: abs.



(at discontinuities, wave length is just too high (i.e. energy just too low) to eject K, L, electrons respectively).

Use H energies:

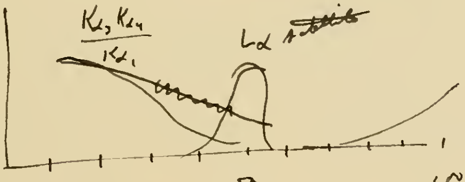
$$W = \frac{1}{\alpha^2}\left[\frac{1}{\sqrt{1 + \frac{\alpha^2 Z^2}{\left(n - j - \frac{1}{2} + \sqrt{(j+\frac{1}{2})^2 - \alpha^2 Z^2}\right)^2}}} - 1\right]$$

$$= -\frac{Z^2}{2n^2} - \frac{\alpha^2 Z^4}{2n^3}\left(\frac{2}{2j+1} - \frac{3}{4}\frac{1}{n}\right).$$

To fit K shells of heavier atoms, must use a screening constant: $(Z - \sigma)$ for Z.

Other lines are observed as well – (usually on short wave length side –

"Satellites":



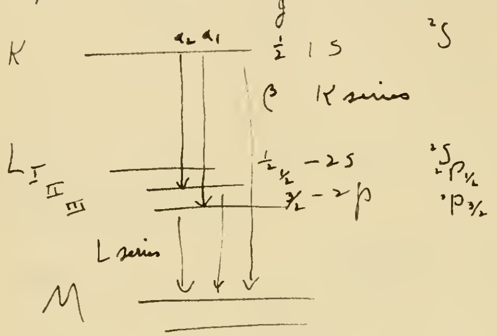
Wentzel assumed ~~satellites~~ due to ionized atoms — but (e.g. both K electrons might be gone, then $KK \to KL$) excitation energy wasn't twice the usual (as it should be on that theory)

X-rays –

X-ray spectra : remove interior electrons :

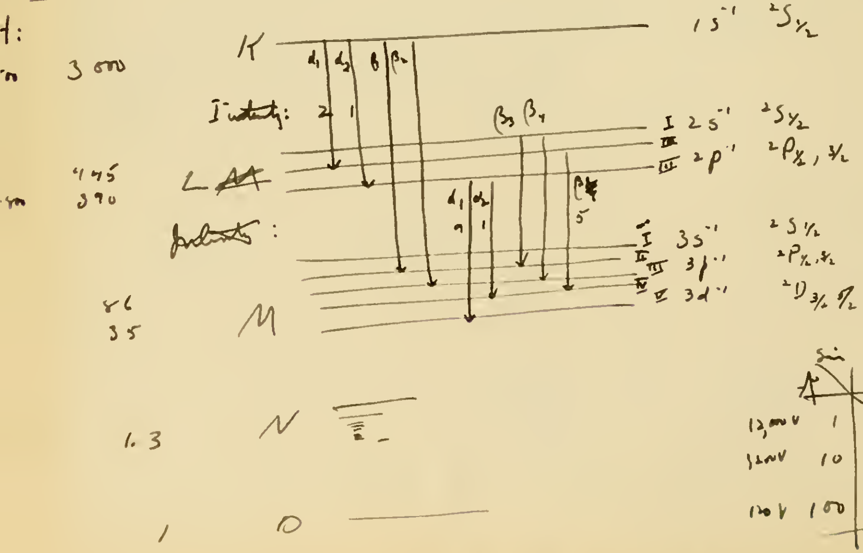
K : 2 : $1s^2$
L : 8 : $2s^2\,2p^6$
M : 18 : $3s^2\,3p^6\,3d$…
N : …
O :

Suppose a K electron missing : high frequency lines from jumps to K shell.



α_1 brighter

5/23/40 | Cadmium : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^6\,4d^{10}\,5s^2$ (K · L, M, N, O) 48



Λ \ Ω	K	L	M	N
13,000 V 1	33	79-90	—	
12 kV 10	12	32	55-65	…
120 V 100	5	15	30	10…

If $S = 0$: $g = 1$ Lorentz case.

$L = 0$, $g = 2$

Pattern same for a given series, since L, S remain the same, hence g the same for all –

(Largest g : $2\frac{2}{3}$;
smallest negative.

C. Strong Field : $\frac{e\hbar}{mc} \sum (L_{z_j} + 2 S_{z_j}) F$ comparable to spin orbit interaction.

Incomplete Paschen-Back Effect:

Field effect >> spin orbit in L-S coupling
>> elect. inter. in j·j "

L-S coupling : L, S quantum no's persist ; but g^2
quantization is spoiled – can use $L S \Lambda \Sigma$

$$W_F = \frac{|e|\hbar}{2mc}(M + \Sigma) F . \qquad \Delta M = 0, \pm 1, \quad \delta\Sigma = 0 .$$

j·j coupling –

D Giant field : >> everything else.

Start with $\prod_{j=1}^{N} n_j l_j \lambda_j \sigma_j$

then $W_F = \frac{|e|\hbar}{2mc} F \left(\underbrace{\sum \lambda_0 + 2 \sum \sigma}_{M + \Sigma} \right)$

Can have L, S quantum no's.

5/21/40 | A. In general $\mathcal{J}_3$ can be diagonal; 1 - M.

$$\Delta M = 0, \pm 1 .$$

$$\pi \quad \sigma$$

B. Weak Field case. Then have $\mathcal{J}$, M.

Now $[\mathcal{J}, (L_3 + 2S_3)] = -i\, \mathcal{J} \times (L_3 + 2S_3)$

$\therefore W_F : g \frac{|e|\hbar}{2mc} F M \qquad g$ a const., Landé factor

line shift: $h\Delta\nu = \frac{|e|\hbar}{2mc} F (g'M' - g''M'')$

$$\frac{|e|\hbar}{2mc} F [(g'-g'')M' - g''(\Delta M)] \qquad \Delta M = M'' - M'$$

if $g' = g'' = g$: $h\Delta\nu = \frac{|e|\hbar}{2mc} g \Delta M$; $\Delta M = 0, \pm 1$,

Triplet.

If we have L-S coupling: $L\, S\, g\, M$

$$\bar{L}_3 = \frac{\mathcal{J}(\mathcal{J}+1) + L(L+1) - S(S+1)}{2\mathcal{J}(\mathcal{J}+1)} M$$

Zeeman ~~perturbation~~ : $-\frac{e\hbar}{2mc} \Sigma (L_{r3} + 2S_{r3}) F$.

$$L_3 + 2S_3 = 2\mathcal{J}_3 - L_3 =$$

$$2M - \bar{L}_3 =$$

$$\therefore g = \frac{2\bar{\mathcal{J}}_3 - \bar{L}_3}{M} = \text{~~2M - g(g+1)~~}\ \frac{2M - \frac{g(g+1) + L(L+1) - \dots}{2g(g+1)}}{M}$$

$$= 2 - \frac{1}{2} - \frac{L(L+1) - S(S+1)}{2g(g+1)}$$

$$= \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2g(g+1)}$$

$$Q_\varepsilon = \sum_{\rho<\tau=1}^{\varepsilon} \left[J_{\tau\rho} - K_{\tau\rho}\, \delta_{s_\rho}^{s_\tau} \right]$$

(same spins).

$$= \frac{1}{2} \sum_{\rho=1}^{\varepsilon} \sum_{\tau=1}^{\varepsilon} \left[J_{\tau\rho} - K_{\tau\rho}\, \delta_{s_\rho}^{s_\tau} \right]$$

Now associate with ψ ~~that~~ ψ' containing the remaining electrons:

For it: $Q_{m-\varepsilon} = \frac{1}{2} \sum_{\rho=\varepsilon+1}^{m} \sum_{\tau=\varepsilon+1}^{m} \left(J_{\tau\rho} - K_{\tau\rho}\, \delta_{s_\rho}^{s_\tau} \right)$ (sum over remaining electrons)

$m = 2(l+1)$

$$Q_\varepsilon + \frac{1}{2} \sum_{\rho=1}^{\varepsilon} \sum_{\tau=\varepsilon+1}^{m} [\quad] = \frac{1}{2} \sum_{\rho=1}^{\varepsilon} \sum_{\tau=1}^{m} \underbrace{\left[J_{\tau\rho} - K_{\tau\rho}\, \delta_{s_\rho}^{s_\tau} \right]}_{B \text{ indep. of } \psi_\rho}$$

$$\therefore Q_\varepsilon = \tfrac{1}{2}\varepsilon B;$$

similarly: $Q_{m-\varepsilon} = \tfrac{1}{2}(m-\varepsilon) B$

$$\therefore Q_{m-\varepsilon} - Q_\varepsilon = \left(\tfrac{1}{2}m - \varepsilon\right) B$$

∴ elect. energies for ψ + associated ψ' differ only by a constant; ∴ same term spacing.

XI Spacing:

1st order ~~multiplet~~ spacing in L.S coupling is just reversed usually larger at his below

ε unit ε missing.

2 — — 0

1 —

0 — = $\frac{1}{2}$

This given by diagonal elements in spin-orbit interaction, using LS SM.

Spin-orbit Matrix in terms of corresponding ψ's = $-$ (matrix terms of ψ's).

Radial ~~integrals~~ identical throughout; (depends only on nl) considering only angular part.

Diagonal elements: $\propto \sum \lambda_i \sigma_i$ for ε electrons.

$$\text{But } \sum_{\varepsilon'} \lambda_i \sigma_i = \sum_{\text{whole shell}} \lambda_i \sigma_i - \sum_{\text{omitted electrons}} \lambda_i \sigma_i = -\sum_{\varepsilon} \lambda_i \sigma_i ,$$

since $\sum_{\text{whole shell}} \lambda_i = 0$.

Non-diagonal elements: occur in product fns differing in only 1 one-electron fn (1st order), + then only when λ differs by 1 -

Can show here also just a reversal in sign.

Spacing of terms (1st order) same for ε present, ε absent, i.e. singlet usually above triplet (at least if only one term of the kind occurs in the configuration) - Then can use diagonal sum rule

Prove diag. elements of elect. inter. is same for ψ & ψ'.

"Holes" in subshells – behave somewhat like single electron (sometimes signs reversed, etc.).

L-S spacing the same; ~~multiplet~~ multiplet spacing reversed –

bigger J's below –

– no. of perm. of $2(2l+1)$ things taken ε at a time $\frac{2(2l+1)!}{?}$

5/18/40 (Sen's notes →)

1. ε electrons outside + closed subshells.

2. ε electrons missing : $2(2l+1) - \varepsilon = \varepsilon'$ present

Same no. of wave f'ns for ε or ε' $= {}^{2(2l+1)}C_{\varepsilon} = {}^{2(2l+1)}C_{\varepsilon'}$

We set up a correspondence ψ for ε electrons $(\lambda_1\sigma_1)\cdots(\lambda_\varepsilon\sigma_\varepsilon)$

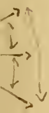
ψ' " ε' omit $(-\lambda_1, -\sigma_1)\cdots(-\lambda_\varepsilon, -\sigma_\varepsilon)$

e.g. $l = 1$: $1\alpha \quad 0\alpha \quad -1\alpha \quad 1\beta \quad 0\beta \quad -1\beta$

× ← present →

$\varepsilon = 2 \quad \psi_{1\alpha\, 0\beta}$

$\varepsilon' = 4$ × ← missing → ×



$\Lambda = \Sigma\lambda$

(since $\Lambda + \Sigma$ same for both; ∴ L S Λ Σ or L S J M f'ns as

for closed shell are zero and $\sum \lambda_i = \sum \lambda_i$) $\Sigma = \Sigma\sigma$ .... same lin. comb. of ψ & ψ'

∴ same ~~terms~~

e.g.: $L_p(\lambda_1\sigma_1) = h_l^{\lambda_1}(\lambda_1+1, \sigma_1)$ if (λ_1+1, σ_1) is missing in ψ

Then $(-\lambda_1, -\sigma_1)$ is absent in ψ', $(-(\lambda_1+1), -\sigma_1)$ is present in ψ'

Then $L_p[-(\lambda_1+1), -\sigma_1] = h_l^{-(\lambda_1+1)}(-\lambda_1, -\sigma_1)$

$= h_l^{\lambda_1}(-\lambda_1, -\sigma_1)$; same term.

1st order energies ; Spin-orbit energy = 0 (since it depends on $\Sigma \lambda\sigma$, which = 0 here)

Elst. $Q = \sum_{\tau=1}^{2l+1} J_{\tau\tau} + 2 \sum_{\tau<\rho=1}^{2l+1} (2J_{\tau\rho} - K_{\tau\rho})$.

$J_{\tau\rho} = \int \psi_\tau^*(q_1)\psi_\rho^*(q_2)\frac{e^2}{r_{12}}\psi_\tau(q_1)\psi_\rho(q_2)$

K

Closed shells + ε electrons in a not closed config : get same ~~multiplicity~~ as if subshells not there ; also same intervals, (grounds so long as no change occurs in subshells

Spin orbit energy = that of the ε electrons –

Elst energy : (a) subshell interaction : shift all levels same amt.
(b) interaction of ε els., as tho they were alone
(c) " " " " with subshells –

c) $\bar{Q} = \sum_{\tau=1}^{2(2l+1)} (2J_{\tau\kappa} - K_{\tau\kappa})$ for single electron $\psi_{n_\kappa l_\kappa \lambda_\kappa \sigma_\kappa}$

+ subshells with $2\nu_0$ els. independent of spin σ_κ ; also indep of λ_κ

Product f'n : $\psi_{subs}\psi_\kappa$; Then $L_p \psi_s \psi_\kappa = h_{l_\kappa}^{\lambda_\kappa} \psi_s \psi_{\kappa+1}$

$$\int \psi_s^* \psi_\kappa^* \sum_{\tau=1}^{2\nu_0} \frac{e^2}{r_{\tau\kappa}} \psi_s\psi_\kappa = \frac{1}{h_{l_\kappa}^{\lambda_\kappa}} \int (L_d \psi_s \psi_{\kappa+1})^* (\Sigma\ ) \psi_s \psi_\kappa$$

$$= \int \psi_s^* \psi_{\kappa+1}^* (\Sigma\ ) \psi_s \psi_{\kappa+1}$$

∴ elst. interaction of ε electrons with closed subshells shift all terms in a config the same – not affecting term spacing

---

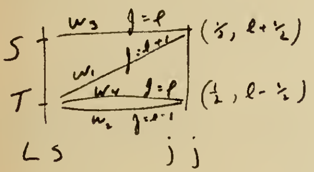
As $\varepsilon \to$ ~~closed~~ closed shell, less f'ns to choose from ; if ε = closed shell − 1, same no. of possibilities as when $\varepsilon = 1$

5/16/40 Intermediate coupling —

$n s \; n' p$.

$m = \lambda$:

ψ_1	ψ_2	ψ_3	ψ_4
$(0\alpha, \lambda-1, \alpha)$	$(0\alpha, \lambda\beta)$	$(0\beta\,\lambda\alpha)$	$(0\beta\,\lambda+1, \beta)$



$$\psi_1' = \frac{1}{D_1}\sqrt{(l+\lambda)(l+\lambda+1)}\;\psi_1 + \frac{1}{D_1}\sqrt{(l+\lambda+1)(l-\lambda+1)}\;\psi_2 + \frac{1}{D_1}\sqrt{(l+\lambda+1)(l-\lambda+1)}\;\psi_3 + \frac{1}{D_1}\sqrt{(l-\lambda)(l-\lambda+1)}\;\psi_4$$

$$D_1 = \sqrt{2(l+1)(2l+1)}\ .$$

ψ_2' . . .

ψ_3'

ψ_4'

$$\Delta W_n = R_{nn} + \sum_{j \neq n} \frac{|R_{nj}|^2}{W_n - W_j}$$

always a repulsive effect;

Closed shells + subshells —

Zero order pert. Theory

Config $nl_{(l\sigma:)}$ $l\alpha, l-1, \alpha \cdots\cdots -l, \alpha$
$l\beta, l-1, \beta \cdots\cdots -l, \beta$

~~Total~~ $2(2l+1)$ el.'s closed sub-shell

For a given n: all full . closed shell .

$\Lambda = \Sigma\lambda = 0$; $\Sigma = \Sigma\sigma = 0$

$\therefore L = 0$, $S = 0$, $J = 0$ $M = 0$ $^1S_0$

e.g. Noble gases — lowest term always ^1S.